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Notes and Comment

Heuristic judgment of mass ratio in two-body collisions

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The logic of judging relative mass from a two-body collision is developed from data presented by Runeson and Vedeler (1993). Data from two experiments are analyzed on a point-by-point basis, and strong support for the theory that mass-ratio judgments are mediated by separate speed and angle heuristics is shown. This analysis is accomplished by reducing the collision event to two elementary features: the presence of ricochet and the ratio of exit speeds. The heuristics that both ricochet and greater exit speed specify relative lightness are shown to explain the basic patterns of data presented by Runeson and Vedeler.

The notion that the optic array contains both variant and invariant structure was developed by Gibson (1979) in laying the groundwork for a theory of affordances in the context of an ecological approach to perception. Basic to Gibson's thinking was that the optic array contains information that a perceiving animal can pick up or resonate to within the purview and limitations of its ecology. In particular, the invariant structure specifies enduring and stable properties of spatial layout. This approach has been primarily distinguished from theories based on inference (Helmholtz, 1910/1962; Rock, 1983) which view perception essentially as problem solving. A test bed for examining these two alternatives has been the perception of relative mass as specified by a collision (Gilden, 1991; Gilden & Proffitt, 1989; Runeson, 1977; Todd & Warren, 1982). The issue here is whether the perception of mass ratio is based on invariant structure in the collision kinematics or on motion cues that are treated inferentially using heuristics.

A primary distinction between the ecological and inferential approaches to the problem of mass-ratio judgment is found in their treatment of error. In the former approach, the presumption is that people are competent at making such judgments and that there is no account for error per se. Early work that was influenced by the ecological movement tended to stress how good people were at making judgments of mass ratio (Kaiser & Proffitt, 1984, 1987). In the inferential approach, the focus is on the pattern of error. If people use heuristics to infer mass ratio, then there will be systematic and egregious errors in some collision regimes. In this regard, Todd and Warren (1982) concluded that mass-ratio judgment was heuristic on the basis of worse-than-chance performance in certain key conditions. Gilden and Proffitt (1989) pursued this work and developed separate angle and speed heuristics that could entirely account for the judgments of mass ratio observed in their studies as well as in those reported by Todd and Warren (1982).

Runeson and Vedeler (1993) have reopened the issue with data from three experiments which they argued were inconsistent with a heuristic framework. The issue, as set forward by Runeson and Vedeler, is that Gilden and Proffitt (1989) reached a number of erroneous conclusions because of errors in their treatment of data and because they held certain crucial parameters constant in their experiments. To be explicit, Runeson and Vedeler argued that it is essential that judgments of mass ratio be understood in the context of threshold determination and that data be represented in terms of psychometric functions. Gilden and Proffitt (1989) represented their data in terms of percentage correct as opposed to the percentage of times a particular object was thought to be heavier. The latter leads to continuous sigmoidal functions, while the former are discontinuous across the point of perceived equality. The second criticism has to do with the fact that Gilden and Proffitt examined the case where one object impinged on another which was initially at rest. Runeson and Vedeler considerd a larger class of initial conditions.

There are other differences between our work and Runeson and Vedeler's that was not made explicit in their article. In both sets of studies, there is particular interest in the role that precollision information plays. In our Experiment 1 and in the studies conducted by Runeson and Vedeler there are occluded conditions where some or all of the precollision information is masked. A key difference here is how occlusion was implemented in the respective studies. Gilden and Proffitt masked only the incoming speed but not the incoming angle, while Runeson and Vedeler completely eliminated the precollision epoch. This difference is important, and the conclusions drawn from our study would certainly not apply to the occluded conditions conducted by Runeson and Vedeler.

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The fundamental disagreement, however, is one of interpretation. We have argued that judgment of mass ratio is heuristic and is informed by postcollision speeds and by the presence or absence of ricochet. Runeson and Vedeler present a Gibsonian interpretation, that these judgments are mediated by complex invariants that refer to an abstract frame of reference in which the vector components are parsed. There is only one way to resolve a dispute over the interpretation of data, and that is to demonstrate the consistency and explanatory power of an interpretation for all relevant data. We accounted for Todd and Warren's (1982) data in our earlier work, and here we show that the heuristic model is consistent in detail with the data presented by Runeson and Vedeler. Our basic claim is that Runeson and Vedeler fail to provide an explanation for any of the important features in their data, and that these features are predicted by a heuristic model. In what follows, we shall attempt to make the following points explicit:

1. It does not make any difference whether judgments of mass ratio are plotted as psychometric functions (percentage of responses that "B" is heavier than "A") or in terms of percentage correct (as Gilden and Proffitt did in their analysis). No conflict in interpretation need arise from this distinction. While plotting response instead of correctness may have greater clarity in displaying the point of subjective equality (50% of responses that "B is heavier"), we have generally, because of our theoretical orientation, been more interested in the error rate; we want to understand why people make errors. In any event, we analyze the data in Runeson and Vedeler as given and convert to error rate as needed.

2. It is true that Gilden and Proffitt did study a restricted domain of collisions. This domain was sufficient to develop and test a two-heuristic theory of mass-ratio judgment. The theory we developed for collisions where one object is initially at rest will be shown to apply to all collisions simulated by Runeson and Vedeler.

3. There is a misunderstanding about what we claimed about the relevance of the precollision kinematics. In the occluded conditions of our Experiment 1, we masked the speed of the incoming ball, but its incoming path was always specified. In the occluded conditions of Runeson and Vedeler, both the speeds and incoming paths were not visible. This difference is critical and, as we shall show, our heuristic model also accounts for the data in Runeson and Vedeler's occluded conditions.

4. The models of data that are constructed by Runeson and Vedeler lack theoretical motivation. The overarching precommitment to explain data in terms of optical invariants is not specific to collision dynamics, and in this case it provides little guidance in model selection. Finally, in the occluded conditions where Runeson and Vedeler have the greatest success in developing models, the heuristic theory is able to make point predictions that are virtually identical with their data.

Heuristics in the Judgment of Mass Ratio

The heuristic theory is based upon specific claims of how collision events are perceptually organized: (1) Trajectories of objects are perceived as being unitary and are not broken down into vector components, and (2) the vector quantity of velocity is perceptually split into angle and speed. Attendant upon this splitting is a theory of judgment that holds that angles and speeds are considered separately in terms of two simple heuristic rules. These rules are: *speed*—the object with the greater postcollision speed is relatively lighter; and *angle*—the one object that ricochets or scatters through the greatest angle is relatively lighter.

Associated with these heuristics is an informal theory of salience. Rates of responding will be influenced by the magnitude of the speed ratio as well as by the distinctive feature of ricochet.

Gilden and Proffitt (1989) arrived at this account on the basis of an experiment (Experiment 2) in which subjects not only judged which of two objects was heavier, but also gave a scaled rating on how much heavier one object was relative to the other. The results from our study, in which one ball struck another which was at rest, were relatively straightforward. When both objects scattered in the forward direction at relatively acute angles ($<45^{\circ}$), ratings were proportional to the ratio of postcollision (exit) speeds, not to the underlying mass ratio. This was evidence for a speed heuristic. The crucial collisions that revealed the existence of an angle heuristic were those in which the impinging object (A) ricocheted backwards but more slowly than the exit speed of the struck object (B). In this case, the speed heuristic informs that B is lighter and the angle heuristic informs that A is lighter. In this conflict situation, our subjects always gave large mass-ratio ratings but were almost equally split in the decision about which object was lighter. This result is important to our argument and bears repeating: all observers were quite convinced that one of the objects was substantially lighter, but they disagreed among themselves over which one it was. This resulted in a bimodal distribution of ratings. In events where A ricocheted backwards and with greater exit speed than B, the heuristics were not in conflict and observers were in virtually unanimous agreement that A was lighter.

Our experiments were motivated in part by the worse-than-chance performance found by Todd and Warren (1982) in the regime where one object impinged on another at rest. On the basis of the highly uneven performance in this condition, Todd and Warren concluded that heuristics might play an important role in the judgment of mass ratio, but they were unable to find a single heuristic that consistently accounted for the data. This was because two heuristics are required to understand performance; one does not suffice. Angle and speed are treated by observers as two separate categories of information, and they require separate heuristics. We have shown that Todd and Warren's (1982) data are completely consistent with a two-heuristic model (Gilden & Proffitt, 1989; see Figure 1 in Gilden, 1991, where this comparison is made explicit). The worse-than-chance performance that Todd and Warren found exists only in the conflict situation when the two heuristics independently inform that both balls are lighter, that is, when the incoming object ricochets but does so with a smaller exit speed.

Partial Replication of Gilden and Proffitt (1989): Experiment 1

In their first experiment, Runeson and Vedeler partially replicated Experiment 1 of Gilden and Proffitt (1989). An object (A) traveled horizontally across the display terminal and struck an object at rest (B). Object B always scattered at an angle of 30° with respect to the horizontal. There were four conditions of occlusion: just A, just B, both A and B, and neither A nor B. When an object was occluded in these experiments, it was not displayed until the postcollision epoch; that is, it spontaneously appeared. The results from the occlusion conditions (both, neither) are replotted in Figure 1 (taken from Runeson & Vedeler's Figure 3). The dependent variable is the proportion of times that subjects responded that object B was heavier. Also shown in Figure 1 are data from our Experiment 1.

In comparing our data with those reported by Runeson and Vedeler, we see that there is excellent quantitative agreement for $m_B/m_A > 1$, but that for $m_B/m_A < 1$ their subjects were perfect while ours had differential amounts of error depending on the occlusion treatment. Although our subjects did sometimes erroneously identify A as being lighter when $m_B/m_A < 1$, the error rates were independent of mass ratio, a trend that is not continued into the $m_B/m_A > 1$ regime.[1] For our purposes here, the important point is that for both data sets, performance is maximal and independent of mass ratio when $m_B/m_A < 1$ and is highly dependent on mass ratio when $m_B/m_A > 1$.

The data set constructed by Runeson and Vedeler in Experiment 1 is a rich source of information about how people perceive mass ratio. There are a number of features of these data that are of interest and which require explanation. They are:

1. When $m_B/m_A < 1$, subjects were highly accurate in deciding which of the two objects was heavier.
2. The point of subjective equality, where m_A is perceived to be equal to m_B, is significantly displaced towards a large mass-ratio $m_B/m_A \approx 1.5$.
3. The judgments improve and approach asymptote on the interval of mass ratio $1.81 < m_B/m_A < 2.69$.
4. Performance in the occluded conditions is not much worse than the conditions in which there was no precollision occlusion.

During the development of the theory of heuristic judgment, we found that having a clear idea of what the collisions look like was essential. For this reason, we have drawn accurate depictions of all collisions discussed in this article. In Figure 2, we illustrate the collisions for Experiment 1. In this figure, all collisions are drawn to the same scale, where arrow length represents speed. The speeds and angles were computed for the exact mass ratios simulated by Runeson and Vedeler, and these values are given in each box. The elasticity in these illustrations is $e = .9$, that used by Runeson and Vedeler.[2]

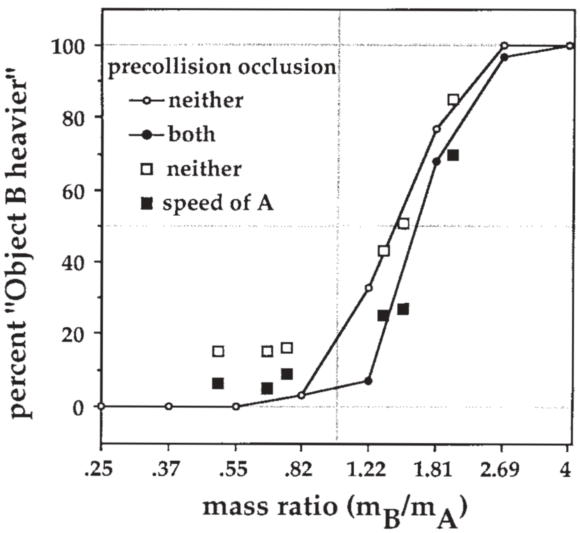

Figure 1. Shown are the percentage of times that, in collisions, subjects chose object B to be heavier than object A as a function of the simulated mass ratio. Open and filled circles are data taken from Figure 3 in Runeson and Vedeler (1993). Open and filled squares are data taken from Figure 3 of Gilden and Proffitt (1989).

A heuristic analysis of perceived mass ratio begins with isolating the speeds in the postcollision epoch and the scattering angle of A. Since the scattering angle of B was constant in this study, it does not provide any useful information. In our theory, people perceptually organize the collision event in terms of the raw trajectories; they do not perceive the collision against the backdrop of some abstract reference frame. Thus, the speeds that we shall be concerned with are the speeds along the trajectories, and the exit angle of A is computed relative to its approach trajectory. The relevant quantities are summarized in Figure 3, where, in panel A, we plot the ratio of postcollision trajectory speeds and, in panel B, the exit angle of object A. The gray lines denote speed equality in panel A and a 90° scattering angle in panel B.

We now turn to a point-by-point analysis of Runeson and Vedeler's data. We confine our comments to their data set because it is identical to ours for $m_B/m_A > 1$, and the difference between theirs and ours for $m_B/m_A < 1$ is a simple main effect of accuracy and not a function of simulated mass ratio. Consider first the occluded condition where both objects are invisible during the precollision epoch but suddenly appear at the moment of collision. In this situation, the basis of judgment will be predominately determined by an analysis of speeds. Fig-

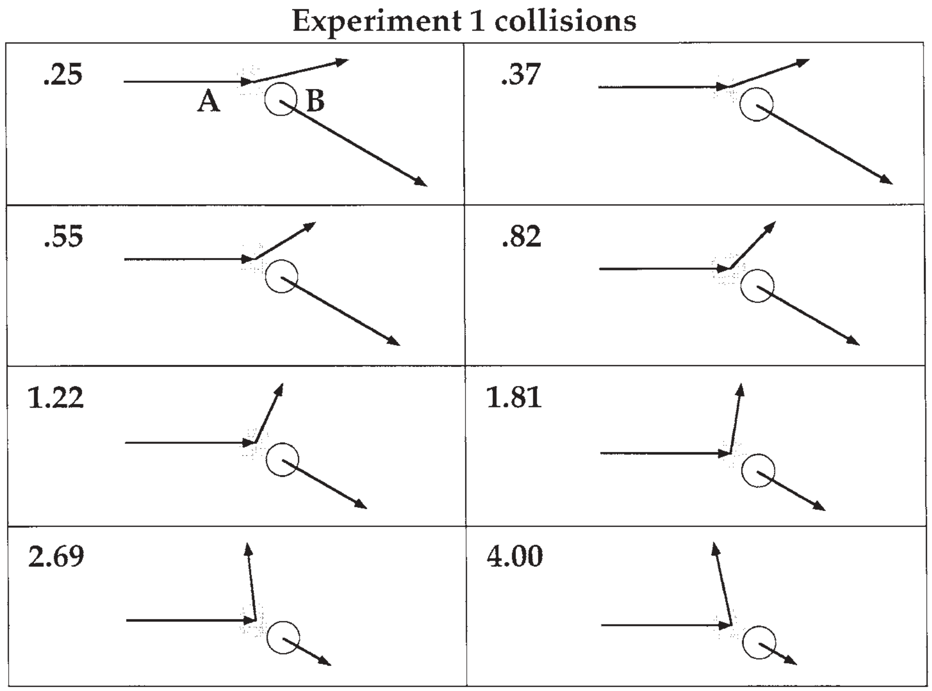

Figure 2. Depictions of the collisions simulated by Runeson and Vedeler (1993) in their Experiment 1. Length of arrow represents speed, and all collisions are drawn to the same scale. The geometry of collisions in this class is such that the initial trajectory of object A was horizontal, object B was initially stationary, and the exit angle of object B was 30°. The elasticity of objects in these simulations was e = .9.

ure 3 illustrates that the speed ratio s_A/s_B is less than unity until $m_B/m_A = 1.99$. In our model, observers judge the faster moving objects as being lighter, and this is reflected in the data by errorless performance for $m_B/m_A < 1$ and by error-ridden performance (about 7% accuracy) at $m_B/m_A = 1.22$. The improved performance at $m_B/m_A = 1.81$ may be due to the near equality of speeds ($s_A/s_B = .92$) as well as to implicit knowledge of the incoming trajectory of A. Blocks of trials with occlusion were mixed with blocks in which A was visible. Since A consistently moved horizontally, observers would naturally refer scattering angles to the horizontal even on trials in which A was not initially visible. Consequently, we would expect performance to be influenced by angle and the scattering angle of A is quite steep for this collision, 81°. Furthermore, if observers do make implicit reference to the horizontal, then we would expect the percentages throughout this condition to be in agreement with the results from our occluded condition in which the horizontal trajectory was manifest—we occluded the initial speed of A, but not its incoming direction. This expectation is clearly realized in the data. Finally, for $m_B/m_A \geq 2.69$, the speed ratio is greater than 1.35, and this is reflected by unanimous agreement that A is lighter.

In the condition where both objects are continuously visible throughout the collision event, the scattering angles are fully specified. For $m_B/m_A < 1$, the scattering angle of A is less than 50°, which we had earlier found (Experiment 2 of Gilden and Proffitt, 1989) is not particularly salient to observers. If both objects scatter forwards at relatively acute angles, observers do not distinguish the quantitative differences in angle as being informative for mass ratio. Consequently, for $m_B/m_A < 1$, observers treat the collisions in terms of the speed heuristic alone, and performance in both occlusion conditions should be identical since the same source of information is used. This expectation is satisfied in Runeson and Vedeler's data.

For $1 \leq m_B/m_A \leq 1.81$, we encounter a regime where exit angle and speed ratio are traded off. Here subjects' preferences reflect the simultaneous approach of the scattering angle of A to 90° and the approach of the speed ratio to unity. These trends make the speed heuristic relatively less viable and the angle heuristic more attractive. The point of subject equality is realized in this interval of mass ratio because it is here that the information informing the heuristics becomes fuzzy. A is not ricocheting, but it is scattering at steep angles. B is scattering faster than A, but not by much. For $m_B/m_A = 1.22$, the scattering angle is 65° but B is moving 50% faster than A. In the absence of ricochet with such a large velocity disparity, we would expect the speed heuristic to be used more often, and 70% of subjects regard A as being heavier. For $m_B/m_A = 1.81$, object B moves only 9% faster than A, which is a difficult difference to discern given that the objects are in different places on the screen and moving in different directions. Under optimal conditions, the Weber fraction for speed discrimination is about 0.1 (McKee, 1981; Orban, Wolf, & Maes, 1984).

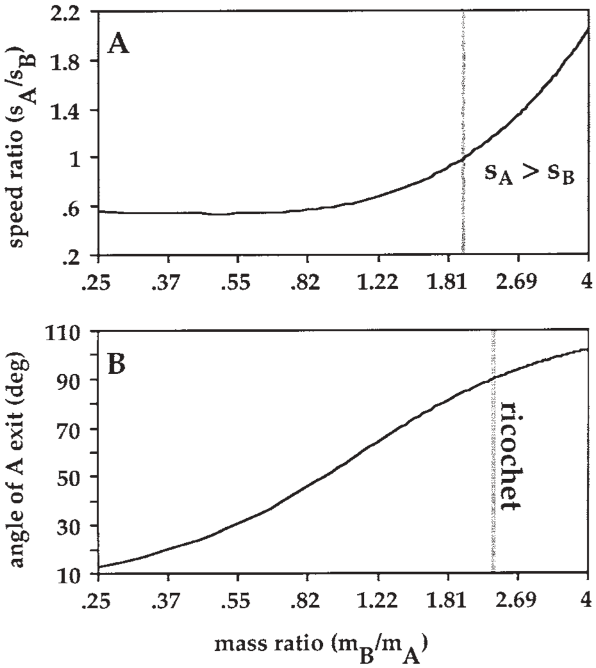

Figure 3. Kinematic quantities associated with the Experiment 1 collisions simulated by Runeson and Vedeler (1993). Panel A shows the speed ratio s_A/s_B as a function of mass ratio. The gray line denotes the mass ratio for which there was exit-speed equality. Panel B shows the exit angle of object A (with respect to the horizontal) for these collisions in this class as a function of mass ratio. The gray line denotes the onset of ricochet for object A.

Given that speed is not useful information here and that the scattering angle of A is quite steep for this collision, 81°, it is not surprising that subjects resort to an angle heuristic even though the trajectory of A is slightly shy of ricochet. Still, more than 20% of responses are that A is heavier.

An interesting question here is whether observers perceive mass ratios in this regime to be near unity, or whether they all think that the one object is definitely heavier—just disagreeing about which one it is. This issue can be decided by collecting ratings of perceived heaviness. A bimodal distribution of ratings on either side of mass equality would favor the interpretation that the point of subjective equality arises from disagreements among subjects. Our earlier work did find bimodal distributions in more extreme cases where A ricocheted and B exited with substantially greater speed.

The final two collisions in this sequence are contained in the interval $m_B/m_A > 1.81$ and are unproblematic. In this interval, A ricochets backwards and it also exits more quickly than B. These are two good reasons for asserting that A is lighter than B, and we see that observers never failed to do so.

We now have an account of the major features of this data set. The displacement of the point of subject equality in mass ratio toward large values of m_B/m_A is apparently due to the displacement of speed equality to $m_B/m_A > 1.8$. If B is moving faster than A, subjects are clearly reluctant to say that B is heavier. The critical collisions that move the point of subjective equality beyond unity are those where the exit angle of A is too acute to be salient and where the speed-ratio heuristic erroneously informs the observer that B is lighter. The difference in performance between the occlusion conditions is essentially centered on the $m_B/m_A = 1.22$ collision. When the incoming trajectory of A is available, observers give the large scattering angle of A weight, leading to relatively more responses that A is lighter.

The critical test bed for our theory of mass-ratio perception requires collisions where A scatters backwards at an exit speed that is slower than that of B. In this case, A ricochets (implying that A is lighter) and B exits more rapidly than A (implying that B is lighter). For collisions with elasticity $e = .9$, where B scatters at 30°, these collisions do not occur. In Runeson and Vedeler's (1993) second experiment, there were several collisions with this property, and it is in this experiment where we find the strongest support for heuristic usage.

Generalization of the Initial Conditions: Experiment 2

Experiment 2 was intended by Runeson and Vedeler (1993) to provide definitive support for their conjecture that observing the precollision kinematics is required for optimal judgment of mass ratio. As their method of occlusion in this experiment was to totally erase the precollision history of both objects, it is not surprising that they found that people were sometimes extremely inaccurate in their judgments in the occlusion conditions. We inquire more generally here into what kind of account may be given to the pattern of heaviness judgments that were obtained. As we will argue, these experiments provide fairly convincing evidence for the heuristic theory originally outlined in Gilden and Proffitt (1989).

In this experiment, both objects were in motion prior to the collision. From a purely formal point of view, this generalization of the initial conditions is not interesting, because there is always a reference frame within which one of the objects is initially at rest. However, this generalization is perceptually relevant, because the collisions do look different and, from a heuristic point of view, how things look is critical. Experiment 2 of Runeson and Vedeler (1993) provides a more general test bed for a heuristic theory than was originally contemplated by Gilden and Proffitt.

In order to better appreciate the collisions simulated in this study, a few comments regarding the physics inherent in a collision may be appropriate. In all studies to date, including this one, the objects are treated as rigid bodies with frictionless surfaces. A consequence of this ideality is that the collisions induce no spin and the collision force acts normal to the object surface at the point of contact. Objects in Runeson and Vedeler's studies, as well as ours, were represented as circles in the plane. In this case, the collision force is transmitted along a line that connects the circle centers and the point of tangency. This line defines an axis that Runeson and Vedeler refer to as the "collision" axis. The orthogonal axis is referred to as the "sweep" axis. As there is no force along the sweep axis, the sweep velocity components are unchanged by the collision. The sweep component is entirely uninformative as to relative mass or any other dynamical quantity and is included simply to make the collision look two-dimensional. Figure 4 illustrates a collision in the frame of reference defined by the collision and sweep axis. This figure will help to define the various conditions that were investigated by Runeson and Vedeler.

In this experiment, the projected speed at which A and B strike on the collision axis was set to a constant of 6.5°/sec, which Runeson and Vedeler refer to as *V*diff. We shall use this as our unit of measurement for speed. There were two speed conditions defined by a parameter referred to as *V*comm. *V*comm is the average of the speeds of A and B as projected onto the collision axis:

$$V\text{comm} = \frac{s_A + s_B}{2},$$

where s_A and s_B refer to the component of velocity along the collision axis.

The two conditions are defined as:

$$\text{Small: } V\text{comm} = .25, s_A = .75, s_B = -.25$$

$$\text{Large: } V\text{comm} = .75, s_A = 1.25, s_B = .25.$$

When *V*comm is small, the objects are directed towards each other, and when *V*comm is large, A catches up to B. Note that in both cases the relative speed of collision ($s_A - s_B$) is unity in these units. The collisions were

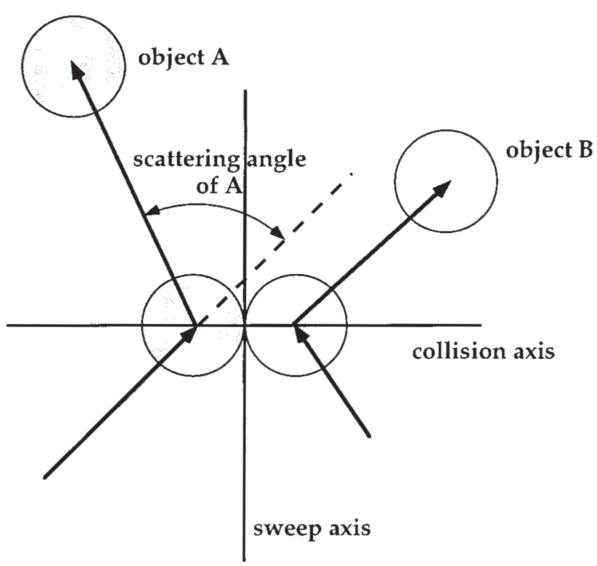

Figure 4. A general collision geometry showing the collision axis and sweep axis. The total scattering angle is defined by the change of heading induced by the collision.

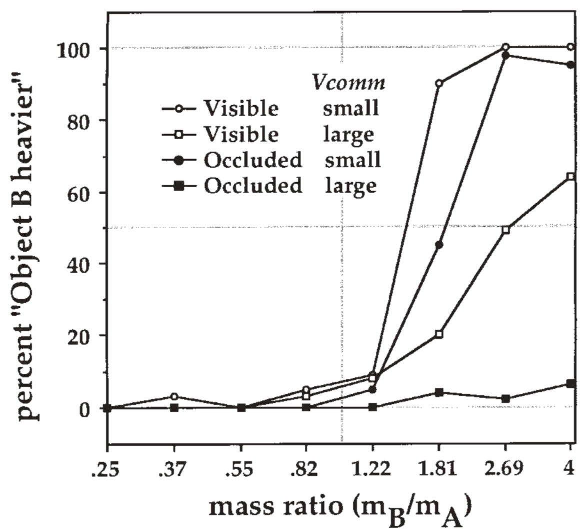

Figure 5. Shown are the percentage of times that subjects chose object B to be heavier than object A in Experiment 2 of Runeson and Vedeler (1993). The relevant velocity and occlusion conditions are explained in the text. These data are taken from Figure 4 of Runeson and Vedeler (1993).

made two-dimensional by adding velocity components orthogonal to the collision axis, along the sweep axis. The exact values of these components were ±.45, and as no forces act along the sweep axis, they were constant throughout the collision. In what follows, we shall denote *V*comm small by B-toward-A and *V*comm large by B-away-from-A in order to have an easily understood way of referring to the different collision conditions.

The data from this experiment (Runeson & Vedeler's Figure 4) are shown in Figure 5 where the percentage of times that object B was rated to be heavier is plotted as a function of mass ratio. There are four conditions of interest generated by crossing the two conditions of occlusion with the two levels of *V*comm. Performance is highly variable over the conditions, and there are, again, a number of features in the data that deserve explanation. In particular:

1. In the B-away-from-A condition, when the precollision epoch is occluded, subjects almost never rate B as heavier.

2. In the B-away-from-A condition, when the precollision epoch is visible, subjects perform more poorly than chance for $m_B/m_A < 2.69$.

3. In the B-toward-A condition, performance is quite similar to that encountered in Experiment 1—highly accurate judgments for $m_B/m_A \leq 1$ and for $m_B/m_A \geq 2.69$, and only a small occlusion effect.

Runeson and Vedeler concluded from these data that performance in the occluded condition (precollision epoch invisible) was so degraded that it made little sense to speak of the perception of relative mass. This observation is not quite correct and is based on averaging the data over the two *V*comm conditions. It is only in the occluded B-away-from-A condition that subjects were

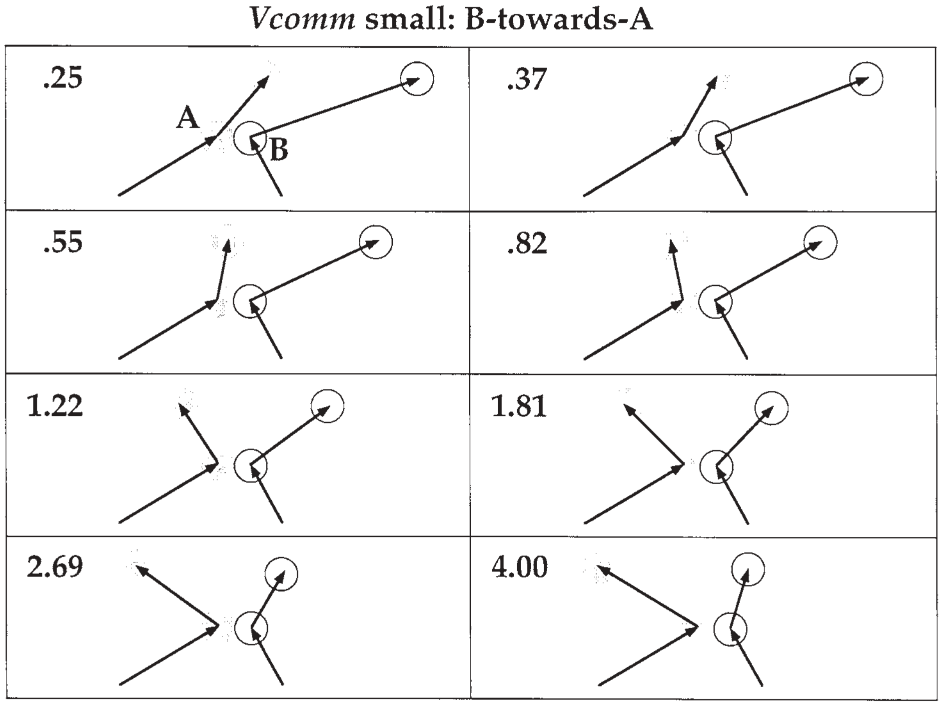

Figure 6. Depictions of the collisions simulated by Runeson and Vedeler (1993) in their Experiment 2 *V*comm small (B-toward-A) condition. Length of arrow represents speed, and all collisions are drawn to the same scale as in Figure 2. The elasticity of objects in these simulations was $e = .9$.

highly inaccurate. In the occluded B-toward-A condition, performance was relatively weak only for the single mass ratio of 1.81. A complete understanding of these data requires that performance in the two *V*comm conditions be individually treated. Averaged data in this context are misleading.

Similar comments apply to the visible conditions. It is particularly odd that in the B-away-from-A conditions, subjects were highly inaccurate when B was heavier than A. Indeed, chance guessing would be an improvement over their performance for $m_B/m_A < 2.69$. The reason for this level of performance must be ascertained, and this cannot be done if the *V*comm conditions are averaged over. In essence, we regard Runeson and Vedeler's analysis of their data to be inadequate, because it does not account for the gross differences that exist between B-toward-A collisions and B-away-from-A collisions.

We have continually stressed in our previous work that it pays to consider what the collisions look like to the observer and so, in Figures 6 and 7, we illustrate the collisions in the *V*comm small (B-toward-A) and *V*comm large (B-away-from-A) conditions. These collisions are drawn on the same scale as in Figure 2, and are depicted so that arrow length represents the actual simulated velocity. All calculations and depictions given here refer to an elasticity $e = .9$ simulated by Runeson and Vedeler. These pictures give an accurate representation, albeit static, of what the collisions looked like to subjects, with the proviso that animated displays had three randomly varied parameters: mirror reversal on the sweep axis, mirror reversal on the collision axis, and orientation of the entire display. These geometric transformations have no effect on the predictions of the heuristic theory.

In Figures 6 and 7, we have not drawn the sweep or collision axes, and we regard these as being irrelevant to the processes whereby people make mass-ratio judgments. In our heuristic theory, observers look at collisions in terms of the trajectories that the objects move along, and not in terms of any projections onto an abstract frame of reference. Thus, when a subject makes a judgment based upon speed, it is the speed along the trajectory that is being used. Furthermore, ricochet is a percept that is always independent of the orientation of a frame of reference. When an object ricochets, it goes backwards relative to its incoming trajectory; no additional frame of reference is necessary for the percept. Although this point of view may appear to be truistic, performance, in fact, would be greatly enhanced if subjects did not look at collisions in terms of trajectories. As Gilden and Proffitt (1989) point out, parsing velocities in terms of their horizontal and vertical components would have led to perfect accuracy in their studies. In contrast to our approach, Runeson and Vedeler posit that people parse collisions in terms of the sweep and collision axes and derive invariants within this abstract coordinate system.

In Figure 8, we recapitulate the collision information essential to a heuristic analysis for both *V*comm conditions. This information is the ratio of trajectory speeds

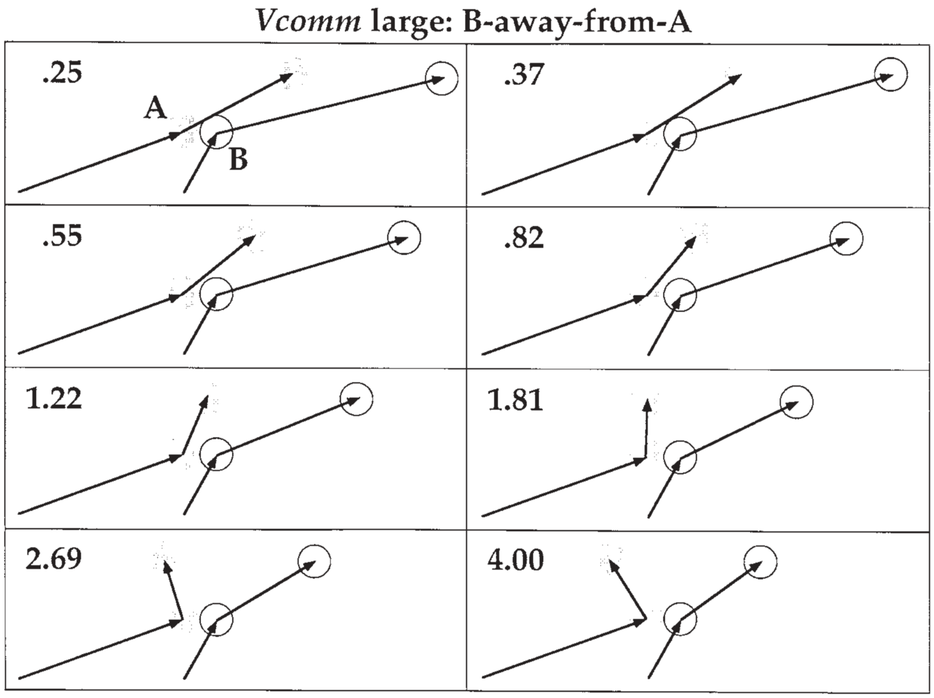

Figure 7. Depictions of the collisions simulated by Runeson and Vedeler (1993) in their Experiment 2 *V*comm large (B-away-from-A) condition. Length of arrow represents speed, and all collisions are drawn to the same scale as in Figure 2. The elasticity of objects in these simulations was $e = .9$.

and the presence of ricochet. Each plot is divided into a number of mass-ratio zones in which distinct heuristic recommendations are made. A summary of these recommendations is given in Table 1, which incorporates the standard logic: (1) following a collision, the lighter object moves faster, and (2) if an object ricochets, it is lighter. We have marked the critical epochs in mass ratio where A or B ricochets and when the trajectory speed of A exceeds that of B. This figure essentially underscores what is already apparent from the collision depictions in Figures 6 and 7.

Division of the mass-ratio interval into heuristic zones will facilitate our account of these data. Consider first the *V*comm small, B-toward-A collisions where the precollision epoch is visible. In zone I, object B ricochets with a greater speed than A. Both heuristics recommend that B is lighter. In zone II, both objects have steep and comparable scattering angles, neither exceeding 90°, and so there is no angle recommendation. In this zone, object B has a larger exit speed than A, by more than 45%, and the speed recommendation continues to be that B is lighter. In these two zones more than 95% of responses were that A was heavier.

In zone III, there is the potential for heuristic conflict because, although A ricochets, its exit speed is slower than B's. However, the one collision tested in this zone occurs at the boundary, $m_B/m_A = 1.22$, where the angles of both outgoing trajectories are close to 90°. Numerically, the exit angles of A and B are 91° and 83°, respectively. This angular difference is quite difficult to discriminate in a collision context where the two objects occupy different parts of the screen and are moving at different velocities. Indeed, it is difficult to discriminate these angles in the static collision diagram in Figure 6, where the angles are explicitly drawn. In cases where both objects appear to ricochet, there clearly can be no angle heuristic recommendation. In this case, the fact that B moves 50% faster than A is decisive and the speed heuristic must be used exclusively. Subjects in this condition respond that B is heavier less than 10% of the time; that is, the speed heuristic induces almost complete incompetence in judging this mass ratio. There were no other collisions simulated in zone III, and we shall return to a discussion of its structure below.

It is instructive to analyze this collision along with the $m_B/m_A = 1.22$ collision in Experiment 1. In the earlier experiment, A had a steep angle of exit (65°) and B also moved about 50% faster than A. However, in Experiment 1, 30% of the responses were that B was heavier; that is, 30% of the responses were influenced by angle. The difference in responding can be traced to the exit angle of B in the two experiments. In Experiment 1, the exit angle of B was 30°, and so there was a noticeable disparity between the two objects. This observation suggests that people may be influenced by large angular differences in rating mass ratio even when ricochet is not present. It is also clear, however, that this is a minor effect since most responses, 70%, are influenced by speed inequality in the absence of ricochet.

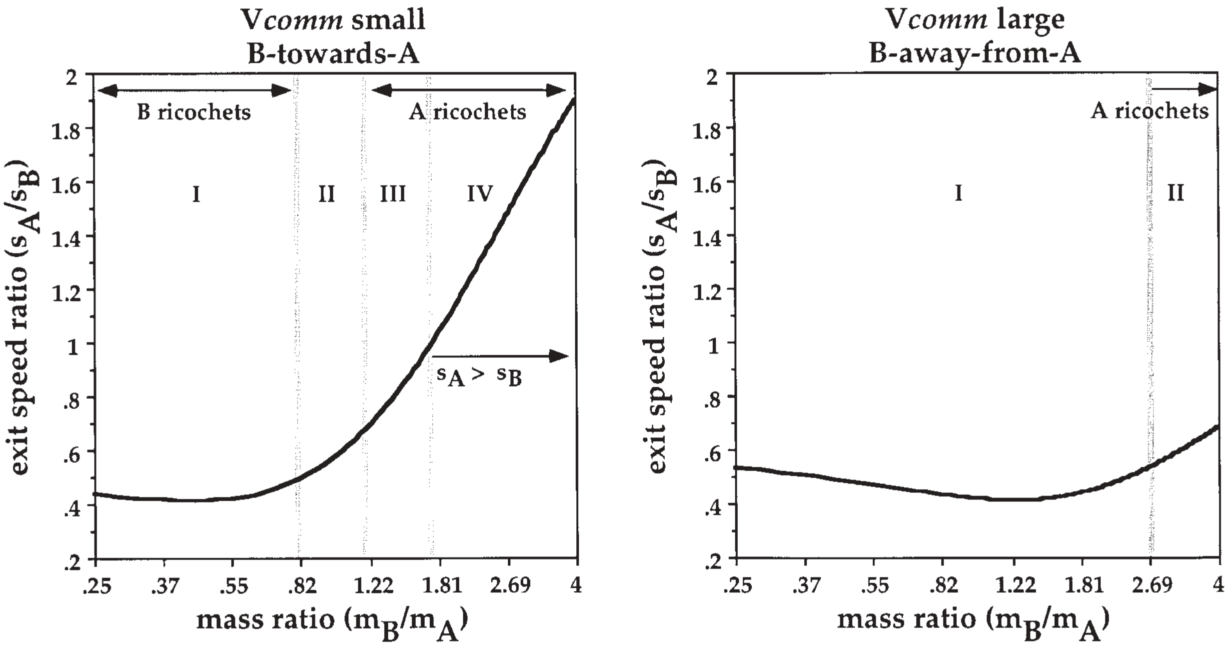

Figure 8. Illustration of the zone structure for collisions simulated by Runeson and Vedeler (1993) in their Experiment 2. Definition of the zones is given in the text and summarized in Table 1.

In zone IV, object A ricochets with a speed greater than or equal to that of B, and so both heuristics recommend that B is heavier. Subjects' responses for collisions in this zone are 100% that B is heavier.

The fact that heuristic conflict occurs in zone III where the psychometric function passes through the 50th percentile has important consequences for an interpretation of the point of subjective equality. Gilden and Proffitt (1989) found that the tradeoff that occurs when heuristics make opposite recommendations is resolved within a dominance metric. That is, people do not average the recommendations of conflicting heuristics, they just opt for one or the other. This type of tradeoff suggests that there is no real point of subjective equality. In zone III, increasing the distal mass ratio will cause an abrupt crossover for each subject individually as the speed heuristic is abandoned in favor of the angle heuristic. Different observers, however, make the crossover at different mass ratios, depending on what they require as evidence for the activation of the angle heuristic. We noted above, for example, that a large postcollision angular disparity might cause some people to base their judgments on angle, while others require ricochet before it occurs to them that angles might be important. Evidence for this picture is the bimodal distribution of perceived heaviness found by Gilden and Proffitt in the case where heuristic recommendations disagreed. Even when a sample population split nearly evenly over which object was perceived to be heavier, every individual was of the opinion that either A or B was much heavier. In this way, a smooth psychometric function may be constructible for a group even when each individual generates a step function in preference. The point of subjective equality in group data is interpreted as the average over the discontinuities within individual preference.

Table 1
Heuristic Analysis of Experiment 2
Which Object Is Heavier?

Condition	Mass-Ratio Interval*	Angle	Speed
*V*comm small	I	A	A
B-toward-A	II	null	A
	III	B	A
	IV	B	B
*V*comm large	I	null	A
B-away-from-A	II	B	A

*See Figure 8 for reference.

Analysis of the B-toward-A occluded condition is more straightforward, because we do not have to consider ricochet as a source of information. Recall that in this condition, the entire precollision epoch is rendered invisible, and therefore there is no sense in which ricochet could be perceived. In this case, a speed heuristic predicts that for $m_B/m_A < 1.8$ A is perceived to be heavier, while for $m_B/m_A > 1.8$ B is perceived to be heavier. The point of subject equality will be the point of speed equality, $m_B/m_A = 1.8$. These predictions are borne out in detail in the data.

There is an obvious aspect to these data which strongly argues for a heuristic interpretation: the data mostly consist of step functions. The A-toward-B visible response function consists practically of two flat pieces. So does the A-toward-B occluded function, except that it also contains a single point of subjective equality. If people had some real sensitivity to the underlying mass ratio, we would expect the responses to have a more continuous nature. Instead, these step-function responses appear to be categorical. On this basis alone, we would suspect that responding was being governed by some sort of discrete process. Heuristics provide exactly the type of discretization required. When we note that there is virtually no variation in the percentage of "B heavier" judgments within any of the marked zones, or between zones where the heuristic recommendations are in agreement, it is apparent that we have discovered the correct heuristics.

The *V*comm large, B-away-from-A, collisions provide an especially sensitive test bed for our heuristic theory, because it is here that we encounter several clear instances of heuristic conflict. In these collisions, both A and B have precollision velocities that are positively directed along the collision axis. In every case, A catches up with B and eventually hits it. Object B, of course, always scatters forwards, but for certain mass ratios, A ricochets backwards. As the speed of B is always greater than that of A, the mass-ratio interval in this experiment divides up rather simply into two zones on the basis of ricochet. Zone division occurs at the point where A scatters at 90°, near $m_B/m_A = 2.6$. Judgments in zone I are dominated by the speed heuristic, but throughout zone II there is heuristic conflict when the precollision epoch is visible.

In zone I, we would expect the speed heuristic to be used regardless of whether or not there was precollision occlusion. For $m_B/m_A \leq 1.22$, subjects almost exclusively rate the faster moving objects as being lighter; less than 10% of responses are that B is heavier. For $m_B/m_A = 1.81$, the influence of angle begins to be seen in the visible condition, and about 20% of responses are that B is heavier. For this collision, the scattering angle of A is about 70°, and we have seen in Experiment 1 (the $m_B/m_A = 1.22$ collision) that the scattering angle may be impressive to observers at this magnitude. The importance of the speed heuristic for this collision should, however, not be minimized. *B is almost twice as heavy as A for this collision, and yet 80% of responses maintain that A is heavier*. This collision alone should create concern for any theory which holds that people have a special sensitivity for perceiving relative mass specified by a collision.

The B-away-from-A zone II collisions present the first clear instances of heuristic conflict in these experiments. Unlike zone III collisions in the B-toward-A condition, here the scattering angle of B is quite shallow, less than 30°, and the ricochet of A is salient and distinct. At the same time, the exit speed of B is at least 50% faster than that of A in this zone. Gilden and Proffitt (1989) found in such circumstances that subjects' impressions of relative mass were bimodally distributed. Analysis of individual responses revealed high internal consistency; subjects did not vacillate in their decisions about a given collision. The disagreement was between subjects. Those who were more impressed by ricochet believed the impinging (A) object was less massive, while those who were more impressed by speed believed the hit (B) object was less massive.

Gilden and Proffitt (1989) did not develop their theory sufficiently to predict the proportion of subjects who will opt for a speed or angle heuristic in any given situation of heuristic conflict. In fact, it is not clear how such a theory could be developed since there is no fundamental theory of angle or speed salience. Here we take zone II data simply as an empirical demonstration of how these heuristics are traded off for various levels of ricochet and angle ratio. For the $m_B/m_A = 2.69$ collision, the scattering angle of A is within a just noticeable difference (jnd) of ricochet, 88°, and the speed ratio is $s_A/s_B = 1.8$. For this combination of parameters, subjects divide up so that they are 50/50 in their decisions of which object is heavier. It is important to stress here that this may not be the point of subjective equality for any individual. As reiterated above, individual choice behavior for similar collisions in Gilden and Proffitt revealed a dominance metric. In the perception of mass ratio, heuristics are not averaged by people.[3]

As the angle of ricochet steepens and the velocity ratio approaches unity, it is sensible that the angle heuristic will be more often used. This trend is evident in the $m_B/m_A = 4.0$ collision, where the scattering angle of A is 102°, the speed ratio is $s_A/s_B = 1.5$, and the percentage of responses that B is heavier reaches about 65%—the largest percentage seen in the study. While our heuristic theory does not predict this number, the bare fact that 35% of responses are in error for this extreme mass ratio is itself telling evidence against any theory of mass-ratio perception that is based on underlying sensitivities to dynamics. The bottom line here is that people are barely above chance for this collision, and this level of performance arises exactly where expected in a heuristic theory—when heuristics make clear and opposite recommendations.

Analysis of the B-away-from-A collisions when the precollision epoch is occluded is trivial. When the incoming trajectories are not visible, the only available heuristic is that slower objects are heavier. Since the speed of A is less than the speed of B for every collision simulated in this condition, our heuristic theory would predict that A would uniformly be judged to be heavier. This prediction is justified by the data.

We regard this set of collisions as providing important confirmatory evidence for the heuristic model of mass-ratio perception. In our earlier work, where B was initially stationary, the set of studied collisions for which there was heuristic conflict was quite small and limited to mass ratios $m_B/m_A < 1.66$. It is gratifying to see this work extended by Runeson and Vedeler in a more general context where the range of mass ratio has been extended. Enlarging the range of mass ratio where heuristic conflict is experienced has permitted the opportunity to evaluate the relative salience of angle and speed information in the resolution of the conflict. We regret that Runeson and Vedeler did not also collect ratings of perceived mass ratio, as this might have clarified whether observers continue to use a dominance metric (all or nothing choice of heuristic) as was found by Gilden and Proffitt (1989).

Construction of Models: Experiment 3

Experiment 3 in Runeson and Vedeler (1993) essentially repeated the conditions of Experiment 2, making minor adjustments to the sweep components in the initial trajectories. Their concern centered on how the collision axis was specified, an issue that we do not regard as relevant. Unfortunately, data from this experiment were presented by Runeson and Vedeler only as averaged over the *V*comm velocity conditions. The analysis we have given to the *V*comm small (B-toward-A) and *V*comm large (B-away-from-A) velocity conditions in Experiment 2 makes it evident that they have radically different kinematics and are treated quite differently by observers. The kinematic distinctions are obvious from a glance at our depictions in Figures 6 and 7 and their summary into a zone structure in Figure 8. The enormous disparity in performance between conditions when the mass ratio exceeds unity is equally clear. Given these distinctions, averaging over the *V*comm conditions in the analysis of data is not tenable. The differences here are real and must be accounted for. There is no way we can analyze averaged data and consequently we refrain from doing so.

Theoretical models were constructed for Experiment 3 data by Runeson and Vedeler on the basis of kinematic information available in the postcollision epoch. It was found that none of the candidate models provided an adequate fit to data in those conditions where both pre- and postcollision kinematics were visible to the observer. They concluded that observers must be using some of the information in the precollision epoch in their judgments of mass ratio. In the occluded conditions, however, there was one model which did have some success. This model was based on the signs and magnitude differences of the postcollision velocity projections onto the collision axis.

It is not surprising that models based solely on postcollision information were not successful in accounting for data in the condition where precollision information was also specified. We have found that observers are very sensitive to ricochet, and it is obvious that this percept requires specification of the initial trajectories. Without some tacit recognition of the initial trajectories incorporated into the model, there is no way for the model to represent ricochet, and therefore no way that the model can account for judgments influenced by ricochet. Any interpretation of our theory which holds that precollision information is irrelevant is simply false: In the heuristic theory, the precollision speeds are not used by observers, but the precollision paths are critical.

A second reason that Runeson and Vedeler's models were unsuccessful in fitting data in the visible conditions may be related to underlying theoretical commitments about what a successful model would look like. Their method of developing models is to isolate single arithmetic functions defined on the kinematic data, and then to test these functions individually. These functions formally are continuous maps of kinematic data into the probability that B is judged to be heavier than A. However, if observers are using a decision theory defined on multiple arguments (speed and angle), as we argue they do in using heuristics, a single continuous arithmetic function may not suffice to explain their data. Recall that Todd and Warren (1982) encountered similar problems when they attempted to fit collision data with a single heuristic. The success that we have had in applying heuristics devolves from allowing the decision theory to incorporate separate arithmetic functions for speed and angle, and by not insisting that both functions are always used in any given decision. Rather, we view the usage of information to be governed by a dominance metric that operates on the relative salience of angle and speed information.

The domain where Runeson and Vedeler did have some success in fitting models to data was in their occluded conditions. From the point of view of heuristic theory, this makes sense, because it is in this regime that a single piece of postcollision information is used—relative exit speed—and a single function defined on kinematic data should suffice. We depart from Runeson and Vedeler here primarily on the issue of what speeds should be used and on the exact form of the function. Runeson and Vedeler use speeds relative to the collision axis. We do not regard the collision axis as being perceptually relevant, and so use the trajectory speeds, the manifest speeds of the objects as they traverse the computer screen. The function studied by Runeson and Vedeler is formed from a speed difference and is continuous on this domain; that is, small changes in speed difference result in small differences in the probability that B is judged to be heavier than A. Our arithmetic function, on the other hand, follows from a discontinuous decision rule on the postcollision speeds: The heavier object moves more slowly, and if the speeds look similar, then the objects have similar masses. The output from such a decision rule is not continuous, but rather is quantized into discrete states. That is, the rule asserts that with unit probability the slower object will be judged to be heavier. Thus there are only three possible outputs in the range of this function: 1 (B moves more slowly), 1/2 (the speed of A and B cannot be distinguished), and 0 (A moves more slowly).

In Figure 9 we illustrate the power of the heuristic theory in its ability to make point predictions in the occluded regime. Data from Experiment 2 are plotted here, as Runeson and Vedeler offer Experiment 3 data only in a form that is averaged over *V*comm. Predictions of the percentage of "B heavier" responses made by the heuristic theory are discretized into the values 0, 1/2, and 1 as required by the decision rule. These predictions are not fits to data. The agreement between the predicted values and the observed values is obvious, and is clearly superior to that displayed by Runeson and Vedeler (see Figure 8 in their paper). We regard this

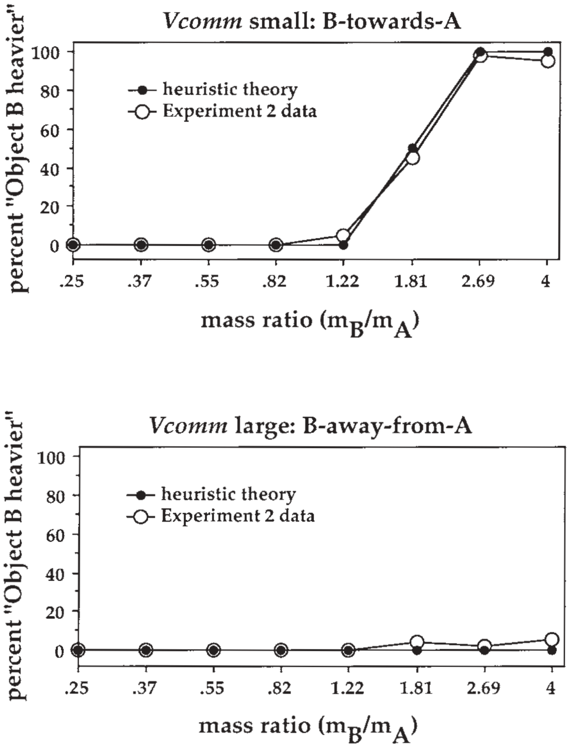

Figure 9. Heuristic point predictions of "percent B heavier" for the occluded collisions simulated by Runeson and Vedeler (1993) in their Experiment 2.

demonstration to be proof of the reality of the speed heuristic.

A final comment on what may be expected from the construction of models in the general case, where the precollision kinematics are specified is in order. A key observation made by Gilden and Proffitt (1989) was that there are regimes of heuristic conflict, regimes where one object ricochets but exits more slowly from the collision site. We found that in this regime there are important individual differences; some people are impressed by ricochet and believe that the one that ricocheted was much lighter, while others are more impressed by disparity in speed and believe that the one that ricocheted was much heavier. These impressions can be manipulated by varying the angle of ricochet and the ratio of exit speeds so that cases arise where the population is near the point of subjective equality (50% of responses are "B heavier"), while no individual believes that the objects have equal mass (recall the bimodal distributions found by Gilden & Proffitt, 1989, in this regime).

Construction of predictive models in regimes of heuristic conflict is very difficult if the goal is to make point predictions. While it is easy enough to isolate where heuristic conflict will occur, it is not possible to predict how any given person will behave, that is, whether they will be dominated by angle or by speed at the given levels of speed ratio and ricochet angle. An empirically oriented theory could be constructed by fitting salience functions, but this is not a terribly informative exercise unless embedded into a more general theory of the perception of angle and speed. In the absence of a theory of salience, the heuristic theory cannot make point predictions in this domain. Yet, the theory is sufficiently well defined to predict that performance in such domains will be poor, possibly worse than chance, even when the mass ratio is quite large. We have seen this very circumstance arise in the *V*comm large, B-away-from-A, condition in Experiment 2, where the point of subjective equality approached $m_B/m_A = 4$. The extreme displacement of the point of subject equality in the exact region where heuristic conflict is predicted to occur is strong evidence for the existence of an angle heuristic and for the usage of a dominance metric in individual choice behavior.

Concluding Comments

The issue of how people judge mass ratio as specified by a collision has bearing on central issues in the theory of perception. The logic of these judgments has become a platform for debating the respective roles of inference and invariance in the extraction of conserved dynamical quantities from proximal stimulation. This debate forms the substance of our interest in collisions, and it dates back to Runeson's (1977) dissertation, where he suggested that Gibson's notion of invariant structure could be extended to the perception of mass ratio. This extension was formalized into a general theory of the specification of dynamics by kinematics (Runeson, 1977; Runeson & Frykholm, 1981, 1983). In our opinion, this is a good idea, well worth investigating. We have, and it does not appear to be correct. Rather, it appears that judgments of mass ratio are mediated by heuristics.

In this article, we have had the opportunity to analyze additional collision data that empirically extend our earlier work. The original account of mass-ratio judgment that we gave in terms of heuristics is supported by these data. By accounting in detail for Runeson and Vedeler's data, we have provided strong evidence for heuristic usage and against the notion that the perception of mass ratio is supported by an apprehension of invariant structure in the optic array. The usage of heuristics is not limited to collision dynamics. In other domains, especially in mechanical systems where rotation is important, we have found little receptivity to the underlying dynamics and evidence for more primitive heuristics having to do with reversal of direction and cessation of motion (Gilden, 1991; Kaiser, Proffitt, Whelan, & Hecht, 1992; Proffitt & Gilden, 1989; Proffitt, Kaiser, & Whelan, 1990).

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NOTES

1. Our Experiment 1 differed from Runeson and Vedeler's Experiment 1 in three key ways. First, they simulated only one approach speed for A, the incoming object, while we simulated two. Secondly, we simulated exit angles for B, the initially stationary object, of 20° and 30°, while they simulated only 30° exits. The additional variation may account for the lower asymptotic performance that was found in our study. Finally, our occlusion conditions did not mask the orientation of the incoming trajectory of A, only its speed. We accomplished this by placing a horizontal gray rectangle along the entry path of A that was tangent to B. Object A appeared at the moment of striking B. The rectangle's height was equal to that of the colliding object and sufficed to specify the orientation of the incoming trajectory while acting as a mask for speed. When Runeson and Vedeler occlude an object they simply do not display it, erasing both path and speed information.

2. In fact, the kinematics of perfectly elastic collisions ($e = 1.0$) are virtually indistinguishable from highly elastic $e = .9$ collisions; speeds are changed by about 5% (the approximate Weber fraction for speed discrimination is 5% to 10%; McKee, 1981; Orban, Wolf, & Maes, 1984) and scattering angles by about 5°.

3. There are numerous situations in which heuristic conflict is resolved within a dominance metric. For example, Shepard (1964a, 1964b) found that in decisions of similarity where two dimensions covaried, people tended to match on one of the dimensions. They did not attempt to trade off differences between dimensions to achieve an average match.