Integrating Information From Separable Psychological Dimensions

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This article examines decision processes in the perception and categorization of stimuli composed of the separable psychological dimensions, orientation and size. The randomization technique (Ashby & Gott, 1988) of general recognition theory, which allows accurate estimation of a subject's decision boundary in a categorization task, is used in 4 experiments. Even though the stimulus components are clearly separable, it was found that Ss were not constrained to use separable response strategies, nor were they constrained to attend to distance to the prototypes. Instead, they used decision rules that were nearly optimal, even if this required information integration or for the Ss to attend to higher level category properties such as component correlation.

A fundamentally important problem is to determine how stimulus dimensions are combined in perceptual processing. Although a variety of perceptual interactions have been proposed (Ashby & Townsend, 1986; Garner & Morton, 1969), it is especially popular to characterize perceptual dimensions according to whether they are integral or separable (Attnavee, 1950; Garner, 1974; Garner & Felfoldy, 1970; Shepard, 1964). A pair of dimensions is said to be integral if it is extremely difficult, if not impossible, to attend to one dimension and ignore the other (Lockhead, 1966). Prototypical integral dimensions are brightness and saturation (e.g., Garner & Felfoldy, 1970; Handel & Imai, 1972; Torgerson, 1958). With separable dimensions, on the other hand, it is easy to attend to one and ignore the other (Shepard, 1964). In fact, it may be very difficult or impossible to integrate information from separable dimensions. Prototypical separable dimensions are hue and shape (e.g., Garner, 1974; Handel & Imai, 1972; Imai & Garner, 1965).

A variety of methods for testing between separability and integrality have been suggested, but perhaps the most popular of these involve speeded classification tasks. Perceptual dimensions are assumed to be integral if a redundancy gain is found with correlated dimensions and if an interference is found when the dimensions are varied orthogonally (e.g., Garner, 1974). If no redundancy gain occurs and no interference is found with orthogonal variation, the dimensions are assumed to be separable.

Ashby and Townsend (1986) identified two different kinds of separability and integrality: perceptual and decisional. A pair of components are perceptually separable if the perceived value of one does not depend on the level of the other. Decisional separability holds if the decision about the level of one component does not depend on the perceived value of the other. A convenient analogy is as follows (Ashby, 1989). Suppose a processing channel exists for each stimulus component. Perceptual integrality occurs if the receptive fields of these channels overlap, whereas perceptual separability is associated with nonoverlapping receptive fields.\(^1\) On the other hand, decisional separability occurs if the subject separately monitors the output of each channel, and decisional integrality occurs if the subject integrates the output of each channel before making a response.

Although they may be empirically correlated, this analogy makes it clear that perceptual and decisional separability are at least logically unrelated. Thus, in any experiment there are four possibilities: (a) both kinds of separability hold; (b) they both fail; (c) perceptual separability fails but decisional separability holds; and (d) perceptual separability holds but decisional separability fails. If we are interested in separability and integrality as perceptual phenomena, then we would like the empirical separability criteria to be met in cases (a) and (d) and to fail in cases (b) and (c).

If both kinds of separability hold, then no redundancy gain will be found in a classification task with correlated dimensions and no interference will occur with orthogonal variation (Ashby & Maddox, 1990; Ashby & Townsend, 1986). If perceptual separability fails, then in general, both a redundancy gain and an interference will be found regardless of whether decisional separability holds. So far, so good. Unfortunately, however, if perceptual separability holds but decisional separability fails, then a redundancy gain or an interference will often occur (Ashby & Maddox, 1990; Ashby & Townsend, 1986). Therefore, the empirical separability criteria are satisfied only in case (a) and so a bias in favor of integrality exists in the popular methods for testing separability. This may be the reason why some researchers have found

\(^1\) Perhaps “sensitivity functions” is more accurate than “receptive fields.” For example, consider stimuli composed of two lines of different orientation. Orientation channels have a bandwidth of 10°-20° (e.g., Thomas & Gille, 1979) and so if the orientations of the two lines differ by less than 10°, they should be perceptually integral. If the orientations differ by more than 20°, they should be perceptually separable.

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evidence of integrality with dimensions that we might expect to be separable. Examples include size and brightness (e.g., Biederman & Checkosky, 1970) and size and orientation (e.g., Garner & Felfoldy, 1970; Smith & Kilroy, 1979).

Because of this bias, it is important for us to assess the empirical correlation between perceptual and decisional separability. Specifically, to what extent are subjects compelled to decisionally separate stimulus components that are perceptually separable? In this article we study the empirical correlation between perceptual and decisional separability with the perceptually separable dimensions of size and orientation. We show not only that subjects are capable of decisionally integrating these components but also that when it is advantageous, they spontaneously integrate the information in nearly optimal fashion.

On the basis of these and other results, we believe that perceptual separability is a property that can be associated with a particular pair of stimulus components and thus is not under the subject's control. If the components are perceptually separable, then the subject is compelled to perceive them separably. On the other hand, decisional separability is not a property that can be associated with particular stimulus dimensions. Instead, decisional separability is an optional strategy the subject may choose to implement during a particular experimental session. The results of this article indicate that the best predictor of whether decisional separability will be satisfied is not the nature of the stimulus dimensions but the characteristics of the experimental design.

The experimental paradigm we use is called the randomization technique and was introduced by Ashby and Gott (1988). This technique makes it possible to estimate accurately the decision rule subjects use during categorization. If we know the decision rule, then it is straightforward to determine whether subjects integrated component information or attended to each component separately (Ashby & Townsend, 1986). In addition to making it easy for us to determine whether decisional separability holds, the randomization technique has several other advantages. First, in the case when decisional separability fails, the technique makes it possible for us to determine the exact nature of the integration process.

For example, we can judge how optimally the subject integrates the component information. A second advantage of the randomization technique is that it allows us to determine whether subjects use deterministic or probabilistic decision strategies. Deterministic responding occurs if a given exemplar always elicits the same response, even when it is almost equally typical of the two categories.

Ashby and Gott (1988) reported a series of randomization experiments with stimuli constructed from a horizontal and vertical line segment joined at an upper left corner. Category membership was determined by the lengths of these two segments. Ashby and Gott found that (a) in most cases, subjects adopted decision rules that were optimal or nearly optimal, even if this required decisional integration or required the subjects to attend to higher level category properties such as component correlation, and (b) subjects displayed a strong tendency toward deterministic responding.

One factor that limits the generalizability of these results is the nature of the stimulus dimensions used by Ashby and Gott (1988). Although Ashby and Gott argued that vertical and horizontal line lengths are perceptually separable, we believe that line lengths may be easier to decisionally integrate than say, hue and shape. Orientation channels in human vision are thought to have a bandwidth of only $10^°$–$20^°$ (e.g., Thomas & Gille, 1979), therefore vertical and horizontal lines should be processed by separate visual channels and should be perceptually separable. Even so, the outputs of these channels are in the same units, namely those of perceived length. Although we do not believe that being in the same units has any bearing over whether a pair of dimensions are perceptually separable, it might make the dimensions very easy to decisionally integrate.

The stimuli used in the experiments reported in this article were semicircles with a spoke that radiates from the center (Shepard, 1964). Examples are shown in Figure 1. The two relevant dimensions are the diameter of the circle and the orientation of the spoke, and thus the underlying perceptual dimensions of size and orientation are in different units. These dimensions have been found to be separable in a number of independent tests (e.g., Burns, Shepp, McDonough, & Wiener-Ehrlich, 1978; Garner & Felfoldy, 1970; Hyman & Well, 1967; Nusofsky, 1986, 1989; Shepard, 1964).

General Recognition Theory

When applied to categorization, general recognition theory (Ashby, 1988, 1989; Ashby & Gott, 1988; Ashby & Perrin, 1988; Ashby & Townsend, 1986) assumes that on each trial, the perceptual effect of a category exemplar can be represented as a point in a multidimensional perceptual space and that repeated presentations of the same exemplar do not always lead to the same perceptual effect. Therefore, the perceptual effects of each exemplar of a category are represented by a multivariate probability distribution. A category is represented perceptually as a probability mixture of the individual exemplar distributions. The theory therefore specifies two different sources of variation: within exemplar and between exemplar. Within-exemplar variation is better known as perceptual noise and is determined by the integrity of the stimulus presentation. Tachistoscopic presentation, low contrast, or masking would all be expected to increase perceptual noise. Between-exemplar variation is determined by the nature of the category and increases with the pairwise dissimilarity of category exemplars. With highly discriminable exemplars presented for long durations at high contrast, within-exemplar variation may often be negligible when compared with between-exemplar variation. In this case, the perceptual representation of each exemplar may be approximated as a point in a multidimensional space and each category as a distribution of exemplar points. This is the situation we focus on in this article.
All of our experiments consist of two categories, both of which contain exemplars of the type shown in Figure 1. The two stimulus dimensions are orientation and size. In each experiment, the exemplars of a category have values on each stimulus dimension that are normally distributed. An example is shown in Figure 2. Category A contains exemplars that tend to have a small size and a large orientation, and Category B contains exemplars that tend to have a large size and a small orientation. Taken together, the size and orientation of an exemplar from either category have a bivariate normal distribution that has a three-dimensional bell-like structure. The height of the bell at any particular size and orientation represents the likelihood that a sample from the category has that particular size and orientation. Rather than draw a three-dimensional figure, it is conventional to depict a bivariate normal distribution by its contours of equal probability, each of which is created by taking a slice parallel to the stimulus plane at some arbitrary height and looking down at the result from above. In Figure 2, the contours of equal probability are circular. Every exemplar corresponding to a point on either circle is equally likely to occur. Note that the diameters of the circles are arbitrary. Larger or smaller circles would occur if either a lower or higher (respectively) slice was taken.

In the randomization technique of general recognition theory (Ashby & Gott, 1988), a pair of bivariate normal distributions like those shown in Figure 2 are numerically specified. Each distribution defines a category, A or B. On trials in which an exemplar from Category A is to be presented, a random sample \((x, y)\) from the A distribution is used to construct a circle of the type shown in Figure 1, with size \(x\) and orientation \(y\). This stimulus is shown to the subject by using high contrast, noise free, response terminated displays. The subject is instructed to respond with the name of the category of which the exemplar is most likely a member. Feedback is given on each trial. Because the distributions overlap, perfect performance is impossible. In fact, we chose the parameters of the distributions (i.e., the means and variances) so that on the average, an optimal responder would be correct between 70% and 80% of the time.

General recognition theory assumes that a practiced subject divides the perceptual space into regions and associates a category label with each region. On each categorization trial the subject determines the region in which the stimulus representation falls and then emits the associated response. Several versions of the theory can be formulated, depending on how the subject divides the perceptual space into response regions. In this article we consider four cases: (a) the independent decisions model; (b) the minimum distance classifier; (c) the general linear classifier; and (d) the optimal model.

Independent decisions models (e.g., Ashby & Gott, 1988; Shaw, 1982; Townsend & Ashby, 1982) assume a two-stage decision process. In the first stage, a separate decision is made about each component (e.g., whether it is large or small, present or absent) and then in the second stage, the results of these decisions are used to select a response. For example, in the Figure 2 example, the subject might decide whether the size is small or large and then whether the orientation is small or large. If a stimulus is presented that is judged to have a small size and a large orientation, the subject would respond A. Note that independent decisions are equivalent to decisional separability. In both cases, the subject’s decision about the level of one component does not depend on the perceived value of the other.

In terms of general recognition theory, independent decisions—and therefore decisional separability—are always identified with decision bounds that are parallel to the coordinate axes. An example is shown in Figure 3a. In this case the exemplars in Category B have more variability on the size dimension than on the orientation dimension. In Category A the amount of variability on each dimension is the same but the size and orientation of the Category A exemplars is positively correlated. In this example, a subject making independent decisions might begin by setting a criterion \(x\) on the size dimension. Note that any exemplar with a size greater than \(x\) will fail to the right of the vertical bound in Figure 3a. The subject next sets a criterion \(y\) on the orientation dimension. Any exemplar with an orientation greater than \(y\) will fall above the horizontal bound. By using these bounds, the subject will respond A to any exemplar falling in the upper left quadrant and B to any exemplar falling in the lower right quadrant. Exemplars falling in either other quadrant contain ambiguous or contradictory information and in these cases, independent decisions models assume that subjects will respond by guessing.

The minimum distance classifier assumes the subject responds with the category that has the nearest (i.e., most similar) mean. This strategy is equivalent to subjects’ using the minimum distance bound (Ashby & Gott, 1988), which in the two-category case is the line that bisects and is orthogonal to the chord connecting the two means. An example of minimum distance classification is shown in Figure 3b. Note that every point above the Figure 3b bound is closer to the A mean and every point below the bound is closer to the B mean.

With normal distributions the mean, the median, and the mode are all equal. It is therefore natural to associate this special point with the category prototype. Another interpretation of minimum distance classification then is that the

![Figure 2](image_url)  
*Figure 2.* Hypothetical marginal distributions and contours of equal probability of the exemplars in two categories.
subject responds with the most similar prototype. Prototype models of categorization (e.g., Homa, Stering, & Trepel, 1981; Posner & Keele, 1968, 1970; Reed, 1972; Rosch, 1973; Rosch, Simpson, & Miller, 1976) can therefore be interpreted as a special case of general recognition theory.

In the general linear classifier, the decision bound is constrained to be linear, but no restrictions are placed on its slope and intercept. General linear classifiers have much more flexibility than minimum distance classifiers (which have none). In this article, we consider the most accurate general linear classifier, that is, the model with the most accurate of all possible linear bounds. An example is shown in Figure 3c. If the variability in any category around the prototype is not uniform in every direction, then the general linear classifier can significantly outperform the minimum distance classifier. In Figure 3c, for example, the general linear classifier predicts higher categorization accuracy than either the independent decisions model or the minimum distance classifier. In one experiment reported by Ashby and Gott (1988) in which the relevant dimensions were both line length, subjects adopted the most accurate linear rule rather than minimum distance classification or independent decisions.

The last version of general recognition theory that we consider is the optimal model, in which the decision bound is placed so that categorization accuracy is maximized. In all but a few special cases, the resulting bounds are nonlinear. For example, with normal distributions, the optimal bounds may be linear, but in general, they are quadratic (e.g., Ashby & Gott, 1988). An example is shown in Figure 3d. The optimal model is important because it sets an upper limit on human performance.

General recognition theory is a theory of experienced categorization. Although it may at times be difficult to determine which decision region an exemplar is in, the theory predicts that once this is established, categorization is essentially automatic. On the other hand, exemplar theories of categorization (Medin & Schaffer, 1978; Nosofsky, 1985, 1986, 1987; Smith & Medin, 1981; Walker, 1975) assume that the subject somehow compares the perceptual representation of each stimulus with the representation of all (or at least a good many) exemplars in each category and then selects a response on the basis of this comparison process. For example, when categorizing faces by race, general recognition theory predicts that subjects have had so much experience with the “face space” that certain races have come to be associated with certain regions of the space and therefore a response can be made without recalling any individual exemplars. In contrast, exemplar theories predict that before a response can be made, the stimulus face must be compared with many individual exemplars of each alternative race.

When one tests between competing categorization theories, the relevant data are the coordinates of each stimulus in the $x$, $y$ stimulus plane and the subject’s responses. After several hundred trials, one need only look at the location of the $A$ and $B$ responses in the stimulus plane to determine what decision rule the subject is using. For example, if the subject is using a simple prototype rule, the $A$ and $B$ responses should be separated by a line that bisects and is orthogonal to the chord connecting the prototypes.

As noted earlier, all of our experiments deal with categories composed of exemplars that are normally distributed on each stimulus dimension. Normal distributions have several properties that we feel are very important when studying categorization. First, Fried and Holyoak (1984) argued that subjects have a natural tendency to assume that categories are normally distributed. Second, normal distributions are mathematically convenient; for example, in such cases, the behavior of the optimal responder is well understood. Finally, normal distributions overlap and the dimensions are continuously valued.
One advantage of overlapping category distributions is their high ecological validity. Natural categories frequently overlap. A person might look like the prototype of one race but be a member of another. A handwritten character might look most like the letter c but have been intended as an a. A musical piece that was written by Vivaldi might sound like Bach. We believe that to understand natural categorization, it is vital to use overlapping categories.

Another advantage of continuous and overlapping category distributions is that they provide a method for rigorously testing between competing categorization theories. This is because there are an unlimited number of exemplars in both categories and each can fall in any region of the stimulus space. Only one bound maximizes categorization accuracy. For example, if the subject uses some bound \( y = g(x) \), then \( y = g(x) \) will be the only curve that perfectly separates the subject's A and B responses in the stimulus plane.

Most categorization experiments use categories with only a small number of exemplars. In this type of design, it is difficult to test between alternative categorization theories because, for example, many bounds (usually an infinite number) perfectly separate the two categories. Therefore, if the subject learns to perfectly categorize the exemplars, it is not possible to determine which bound was used. Sometimes, after perfect accuracy is achieved during a learning phase, a novel exemplar (the transfer stimulus) is presented to the subject. The subject's response is used to rule out some of the competing categorization theories. Many possibilities cannot be ruled out in this fashion, but more important, even if successful, this strategy only indicates which of the many acceptable rules was easiest to use during learning. Because there are many equally accurate bounds, the experimenter is, in essence, telling the subject to choose whichever one requires the least effort to implement. This is an interesting research question, but we do not believe that it reveals much about natural constraints on human pattern classification. Just because a subject chooses to use some simple categorization strategy does not mean that the same subject cannot learn more complex strategies or even that such simple strategies dominate everyday categorization.

The four experiments discussed in this article are described by the contours of equal probability illustrated in Figure 4. They were designed to test the ability of subjects to integrate stimulus components that are clearly separable. In Experiments 1 and 2, decisional separability was optimal. We ran these experiments as a control to ensure that the randomization technique is able to identify a separable strategy and also to get some idea of the amount of response variability to expect from uncontrolled perceptual and criterial noise. In Experiment 3, the optimal rule required integration, but it only increased accuracy 7% above the separable strategy. Minimum distance and general linear classifiers made the same predictions in Experiment 3. In Experiment 4 the optimal rule again involved integration, but this time minimum distance classification predicted suboptimal performance. These four experiments used the same experimental paradigm and so the general methods used in each are described first and additional details given as each experiment is discussed.

Figure 4. Schematic illustration of the contours of equal probability associated with the distributions of (a) Experiments 1 and 2, (b) Experiment 3, and (c) Experiment 4. (In each case, the solid line indicates the optimal decision bound.)
General Method

Subjects

All subjects in Experiments 1, 3, and 4 were volunteers from the University of California, Santa Barbara (UCSB), community who were paid a base rate of $4.00 for each 1-hr experimental session plus a bonus of $0.10 for every percentage point correct above 70%. Subjects in Experiment 2 were members of a UCSB introductory psychology course who participated in the experiment as partial fulfillment of their course requirements. All subjects had 20/20 vision or vision corrected to 20/20. Six subjects participated in Experiment 1, 8 in Experiment 2, 5 in Experiment 3, and 4 in Experiment 4. No subject participated in more than one experiment.

Stimuli

Each stimulus was a semicircle of varying radius with a line segment projecting from its center to the edge and varying in angle of rotation. The stimuli were computer generated and displayed on a Mitsubishi color display monitor Model No. C-9918NB in a dimly lit room. In every experiment two categories, A and B, were each created by defining a specific bivariate logistic distribution. The logistic was chosen because of its similarity to the normal distribution, but it has a simple closed-form expression for its cumulative distribution function.

On each trial, stimulus generation proceeded as follows. First, a category (A or B) was randomly selected (each with probability .50). Next, a random sample $(x, y)$ was drawn from the appropriate bivariate logistic distribution. A stimulus was then constructed with radius $x$, and angle of rotation $y$.

In Experiments 1 and 2, the parameters of the bivariate distributions were selected so that the optimal responder could correctly classify the samples with probability .73 and in Experiments 3 and 4 with probability .80.

Procedure

On every trial, the subject’s task was to categorize the stimulus as an exemplar of Category A or B by pressing an appropriate button. Subjects were told that even an “expert” would make frequent errors and that they would receive a bonus for every percentage point correct above 70%. Accuracy was stressed much more than speed. The stimulus display was terminated either by the subject’s response or after 5 s if the subject had not responded. Feedback showing the correct response appeared on the screen immediately following the subject’s response. There was a 3-s pause between trials.

The first 100 trials of each session served as practice. A pause separated the practice from the 300 experimental trials. During the pause the subject was allowed to ask questions about the procedure. Before the practice block, each prototype (i.e., the means of each distribution) was displayed alternately with its category label five times. The experimental session consisted of four 75-trial blocks. There was a 30-s pause between blocks to allow subjects to rest.

Experiment 1

Experiment 1 had two goals. The first was to verify that subjects would use decisional separability when separability was optimal and when the stimuli were constructed from components that had been found to be perceptually separable. When the two dimensions were line length, Ashby and Gott (1988) found that subjects had difficulty adopting a separable strategy even when separability was optimal. The second goal was to determine the amount of response variability that occurs in the randomization technique when the stimulus dimensions are size and orientation.

The contours of equal probability that describe the Experiment 1 category distributions are shown in Figure 4a. Table 1 gives the exact parameter values describing the populations.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$\mu_a$</th>
<th>$\mu_B$</th>
<th>$\sigma_x$</th>
<th>$\sigma_y$</th>
<th>$\text{COV}_{xy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiments 1 and 2, size constant</td>
<td>225</td>
<td>250</td>
<td>42</td>
<td>42</td>
<td>0</td>
</tr>
<tr>
<td>Experiments 1 and 2, orientation constant</td>
<td>225</td>
<td>225</td>
<td>42</td>
<td>42</td>
<td>0</td>
</tr>
<tr>
<td>Experiment 3</td>
<td>200</td>
<td>250</td>
<td>42</td>
<td>42</td>
<td>0</td>
</tr>
<tr>
<td>Experiment 4</td>
<td>250</td>
<td>200</td>
<td>42</td>
<td>42</td>
<td>0</td>
</tr>
</tbody>
</table>

Note. 250 orientation units = $\pi/4$ radians; $\mu_a$ = mean on the size dimension; $\mu_B$ = mean on the orientation dimension; $\sigma_x$ = standard deviation on the size dimension; $\sigma_y$ = standard deviation on the orientation dimension; and $\text{COV}_{xy}$ = size–orientation covariance.

To actually compute optimal bounds, it is necessary to equate size units with orientation units. Circle size was measured in arbitrary screen units. The screen resolution was $1,024 \times 768$ pixels. Orientation was measured in radians. There is no a priori reason to expect one screen unit to psychologically equal one radian. When calculating optimal bounds, we arbitrarily assumed that one quarter of a semicircle (i.e., $\pi/4$ radians) was psychologically equal to about one quarter of the screen width (i.e., 250 units).

Even so, the randomization technique does not require a close correspondence between the stimulus and the perceptual spaces. The only requirement is that each stimulus and perceptual dimension be monotonically related (as both Stevens's and Fechner's law predict). In this case, independent decisions bounds in the perceptual space will show up as independent decisions bounds in the stimulus space, and optimal bounds...
in the perceptual space will show up as optimal bounds in the stimulus space. With the stimuli used in the experiments reported in this article, however, we expect the stimulus and perceptual spaces to be closely related. First, the Stevens' exponent for length (i.e., diameter or size in this case) is very close to 1.0 (Stevens, 1961) and so the perceived size dimension should approximately equal the physical size dimension. Second, psychological scaling solutions that have been derived for these stimuli have found the perceptual space to be very similar to the stimulus space (Nosofsky, 1985, 1986, 1987; Nosofsky, Clark, & Shin, 1989; Shepard, 1964). The angles from the stimuli used in these studies have ranged from 25° to 155°. Ninety-five percent of our stimuli (i.e., ± 2 standard deviations) had angles that ranged from 21° to 60°.

Results and Discussion

The randomization technique depends on stimuli that are constructed by repeated random sampling from a pair of probability distributions. Figure 4 describes the stimulus populations. During the course of a 50-min experimental session, a subject may be presented a stimulus sample that is uncharacteristic of the population. It is important for us to verify that the stimuli actually presented to a subject conform reasonably well to the population values. Table 2 describes the characteristics of the sample stimuli actually presented to the subject during his or her last experimental session.

First, for each subject we computed the percentage of exemplars correctly classified by the optimal bound. This is the percentage correct we would expect from a subject perfectly using the optimal (population) bound. The stimulus distribution parameters were selected so that in the population this value should equal 73%. From Table 2 we see that these values ranged from 72% for Subject 1 in the size constant condition to 80% for Subject 1 in the orientation constant condition.

As a comparison, we also determined the linear bound that best separated the exemplars from the two categories. This is the bound that minimizes the sum of the squared categorization errors (SSE), where a categorization error is defined as the distance from an incorrectly categorized exemplar to the decision bound. The bounds shown in Figure 4a best separate the exemplars in the two stimulus populations. Table 2 gives the equations of the bounds that best separated the sample of exemplars actually presented to each subject during his or her last experimental session. Note that the absolute value of the slopes are all less than .45. More important perhaps, note that none of these bounds predict a significantly higher accuracy than the optimal population bound. The stimulus samples actually presented to the subjects during their last experimental session therefore seem representative of the Figure 4 populations.

Figure 5 shows the responses of typical subjects during their last experimental session. Figure 5a shows the responses of Subject 1 in the orientation constant condition, and Figure

<table>
<thead>
<tr>
<th>Subject</th>
<th>% of exemplars accounted for by the optimal bound</th>
<th>% of exemplars accounted for by the bound ( x = 250 )</th>
<th>Best-fitting linear bound</th>
<th>% of exemplars accounted for by the independent decisions bounds</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Equation</td>
<td>SSE</td>
<td>%</td>
</tr>
<tr>
<td>Experiment 1, size constant condition</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>72</td>
<td>—</td>
<td>( y = -0.05x + 237 )</td>
<td>86,591</td>
</tr>
<tr>
<td>2</td>
<td>74</td>
<td>—</td>
<td>( y = 0.20x + 173 )</td>
<td>107,814</td>
</tr>
<tr>
<td>3</td>
<td>73</td>
<td>—</td>
<td>( y = -0.04x + 240 )</td>
<td>72,119</td>
</tr>
<tr>
<td>Experiment 1, orientation constant condition</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>80</td>
<td>—</td>
<td>( y = -0.45x + 341 )</td>
<td>59,580</td>
</tr>
<tr>
<td>2</td>
<td>77</td>
<td>—</td>
<td>( y = -0.38x + 325 )</td>
<td>82,953</td>
</tr>
<tr>
<td>3</td>
<td>75</td>
<td>—</td>
<td>( y = -0.05x + 235 )</td>
<td>149,993</td>
</tr>
<tr>
<td>Experiment 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>84</td>
<td>—</td>
<td>( y = 1.46x - 297 )</td>
<td>61,276</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
<td>—</td>
<td>( y = 1.08x - 18 )</td>
<td>48,344</td>
</tr>
<tr>
<td>3</td>
<td>83</td>
<td>—</td>
<td>( y = 1.07x - 14 )</td>
<td>50,082</td>
</tr>
<tr>
<td>4</td>
<td>81</td>
<td>—</td>
<td>( y = 1.12x - 27 )</td>
<td>60,520</td>
</tr>
<tr>
<td>5</td>
<td>81</td>
<td>—</td>
<td>( y = 0.87x + 21 )</td>
<td>51,030</td>
</tr>
<tr>
<td>Experiment 4</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>80</td>
<td>66</td>
<td>( y = 0.95x + 12 )</td>
<td>13,728</td>
</tr>
<tr>
<td>2</td>
<td>76</td>
<td>65</td>
<td>( y = 0.97x + 6 )</td>
<td>14,991</td>
</tr>
<tr>
<td>3</td>
<td>86</td>
<td>63</td>
<td>( y = 0.98x + 4 )</td>
<td>8,747</td>
</tr>
<tr>
<td>4</td>
<td>84</td>
<td>67</td>
<td>( y = 0.88x + 25 )</td>
<td>19,705</td>
</tr>
</tbody>
</table>

Note. SSE = Sum of squared categorization errors.
with the percentage of exemplars accounted for by the optimal bound from Table 2 indicates that the accuracy of half the subjects was within 5% of optimal. Two other subjects were within 10% and only Subject 3 in the orientation constant condition was more than 10% below the optimal level.

Table 3 also describes the linear bound that best separated each subject's responses, the SSE, and the percentage of responses accounted for by this bound, as well as the SSE and percentage of responses accounted for by the optimal bound. When fitting response data, the SSE of a bound is the sum of squared categorization errors, where now a categorization error is defined as the distance from the bound to an exemplar for which the subject emitted an incorrectly predicted response. Note that for 4 subjects, the absolute value of the slope of the best-fitting bound was less than .15. In each of these cases, the optimal bound did almost as well as the best-fitting bound in predicting the subject's responses. Therefore, we can conclude that these subjects effectively ignored the irrelevant dimension.

For Subject 2 in the size constant condition and for Subject 3 in the orientation constant condition, the absolute value of the slope of the best-fitting bound was greater than .30 (see Table 3). In addition, for both of these subjects the optimal bound accounted for more than 5% fewer responses than the best-fitting bound. This indicates that at least to some extent, these subjects integrated information from the two stimulus components. Ashby and Gott (1988) found that when the stimulus dimensions were in the same units (horizontal and vertical line length), most subjects had difficulty ignoring an irrelevant dimension. Our results indicate that some subjects experienced this difficulty even when the stimulus dimensions are in different units (orientation and size). Such pronounced individual differences also underscore the importance of single-subject analyses when the randomization technique is used.

Experiment 2

There are several reasons why some subjects might have had difficulty responding separately in Experiment 1. First, in spite of the fact that the components are perceptually separable, some subjects might actually be constrained to respond with an integral strategy. Another possibility is that because the task is so difficult (i.e., even the ideal observer will make many errors), some of our subjects had not yet learned that decisional separability was the optimal response strategy. Experiment 2 tests these two hypotheses by replicating the experiment with subjects who have been shown the optimal rule and who have been instructed to use it. For example, in the orientation constant condition, the subjects were told to imagine a figure with a diameter midway between the diameters of the two prototypes. They were then told that on each trial they should compare the diameter of the stimulus with this imagined referent and that they should respond A if the stimulus had a greater diameter and B if it had a smaller diameter. Eight subjects each completed one 50-min experi-

---

2 This idea was suggested by Rob Nosofsky (personal communication, March 16, 1989).
Table 3  
Characteristics of the Experimental Responses

<table>
<thead>
<tr>
<th>Subject</th>
<th>% correct</th>
<th>Optimal % correct</th>
<th>Best-fitting linear bound</th>
<th>Optimal bound</th>
<th>$x = 250$ bound</th>
</tr>
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<tbody>
<tr>
<td></td>
<td></td>
<td>Equation</td>
<td>SSE</td>
<td>%</td>
<td>SSE</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experiment 1, size constant condition</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>67</td>
<td>$y = -0.04x + 245$</td>
<td>7,167</td>
<td>88</td>
<td>11,436</td>
</tr>
<tr>
<td>2</td>
<td>69</td>
<td>$y = -0.31x + 287$</td>
<td>13,882</td>
<td>86</td>
<td>19,935</td>
</tr>
<tr>
<td>3</td>
<td>64</td>
<td>$y = -0.08x + 240$</td>
<td>69,959</td>
<td>76</td>
<td>163,248</td>
</tr>
<tr>
<td>Experiment 1, orientation constant condition</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>73</td>
<td>$y = -0.05x + 222$</td>
<td>30,745</td>
<td>82</td>
<td>35,285</td>
</tr>
<tr>
<td>2</td>
<td>73</td>
<td>$y = -0.13x + 251$</td>
<td>25,325</td>
<td>85</td>
<td>27,133</td>
</tr>
<tr>
<td>3</td>
<td>62</td>
<td>$y = -0.03x + 385$</td>
<td>112,916</td>
<td>71</td>
<td>145,695</td>
</tr>
<tr>
<td>Experiment 2, size constant condition</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>71</td>
<td>$y = 0.39x + 134$</td>
<td>34,286</td>
<td>79</td>
<td>43,413</td>
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<tr>
<td>2</td>
<td>66</td>
<td>$y = -0.06x + 238$</td>
<td>19,577</td>
<td>84</td>
<td>19,883</td>
</tr>
<tr>
<td>3</td>
<td>74</td>
<td>$y = -0.4x + 229$</td>
<td>31,926</td>
<td>93</td>
<td>3,819</td>
</tr>
<tr>
<td>4</td>
<td>65</td>
<td>$y = -0.2x + 276$</td>
<td>16,554</td>
<td>83</td>
<td>20,088</td>
</tr>
<tr>
<td>Experiment 2, orientation constant condition</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>72</td>
<td>$y = -0.08x + 227$</td>
<td>6,942</td>
<td>88</td>
<td>16,590</td>
</tr>
<tr>
<td>2</td>
<td>70</td>
<td>$y = -0.06x + 236$</td>
<td>20,950</td>
<td>86</td>
<td>21,867</td>
</tr>
<tr>
<td>3</td>
<td>74</td>
<td>$y = -0.1x + 252$</td>
<td>11,965</td>
<td>88</td>
<td>12,901</td>
</tr>
<tr>
<td>4</td>
<td>67</td>
<td>$y = -0.06x + 236$</td>
<td>11,004</td>
<td>87</td>
<td>11,671</td>
</tr>
<tr>
<td>Experiment 3</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>77</td>
<td>$y = 0.15x + 209$</td>
<td>33,829</td>
<td>84</td>
<td>95,081</td>
</tr>
<tr>
<td>2</td>
<td>70</td>
<td>$y = 0.33x + 160$</td>
<td>29,467</td>
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<td>60,264</td>
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<tr>
<td>3</td>
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<td>$y = 0.45x + 130$</td>
<td>6,798</td>
<td>88</td>
<td>17,647</td>
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<tr>
<td>4</td>
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<td>$y = 0.53x + 111$</td>
<td>9,843</td>
<td>87</td>
<td>20,057</td>
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<tr>
<td>5</td>
<td>74</td>
<td>$y = 0.21x + 183$</td>
<td>32,424</td>
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<td>51,736</td>
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<tr>
<td>Experiment 4</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>69</td>
<td>$y = 0.84x + 47$</td>
<td>24,591</td>
<td>74</td>
<td>30,303</td>
</tr>
<tr>
<td>2</td>
<td>65</td>
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<td>35,946</td>
<td>68</td>
<td>37,913</td>
</tr>
<tr>
<td>3</td>
<td>72</td>
<td>$y = 0.98x + 10$</td>
<td>30,336</td>
<td>70</td>
<td>31,623</td>
</tr>
<tr>
<td>4</td>
<td>55</td>
<td>$y = 0.76x + 65$</td>
<td>52,250</td>
<td>63</td>
<td>72,960</td>
</tr>
</tbody>
</table>

Note. SSE = Sum of squared categorization errors.  
$P^*$ = probability that the subject responds optimally to any stimulus that falls in the upper right or lower left quadrants of the stimulus space.

mental session. In all other respects, Experiment 2 is identical to Experiment 1.

If some subjects still have trouble decisionally separating the component information, then it cannot be because they have failed to learn the optimal rule. Instead, it must be because they are constrained by decisional integrality.

Results and Discussion

Because subjects were told exactly what decision rule to use, the efficacy of the stimulus generation procedure is not as crucial in this experiment as in the others reported in this article and so we omit a detailed analysis of the stimulus samples in this case.

Table 3 outlines characteristics of the responses given by the subjects during their only experimental session. Comparing columns 2 and 3, we see that the accuracy of each subject was within 8% of what they would have achieved had they used the rule that we described to them perfectly. This performance is slightly better than what we observed in Experiment 1. There, 4 of the 6 subjects were within 8% of the optimal responder's predicted accuracy. The other 2 subjects were within 13%. Next, note that for 6 subjects, the absolute value of the slope of the best-fitting linear bound was less than (or equal to) .1. For one subject it was .2 and for another it was .39. However, before concluding that these latter two subjects integrated the component information, compare the percentage of responses accounted for by the best-fitting bound and by the optimal bound. For both of these subjects, the discrepancy was no more than 2%. This suggests that the optimal bound accounted for the data almost as well as the best-fitting bound. We conclude that all subjects in Experiment 2 were successful at decisionally separating the component information. Therefore, it appears that the trouble the 2
subjects in Experiment 1 had in adopting decisional separability occurred because they had not yet learned that decisional separability was optimal and not because they were constrained to integrate the information.

The results of Experiment 2 can also be used to help determine whether the subjects in Experiment 1 used deterministic or probabilistic decision rules. The decision process is deterministic if, on every trial, subjects base their response on a rule like "if \( x > x_0 \), then respond A. If \( x < x_0 \), then respond B." Variability in \( x \) is called perceptual noise and variability in \( x_0 \) is called criterial noise. Even if both sources of noise are present, the rule is still deterministic in the sense that for given values of \( x \) and \( x_0 \), if \( x > x_0 \), then response A is emitted with probability 1.0. If \( x > x_0 \), but the probability of responding A is between 0 and 1.0, then the decision rule is probabilistic. Many popular categorization theories postulate probabilistic decision rules (e.g., Medin & Schaffer, 1978; Nosofsky, 1986, 1987, 1989). Ashby and Gott (1988) found strong evidence of deterministic responding.

Subjects in Experiment 2 were told to use a deterministic rule. Therefore any responses not accounted for by the best-fitting bound should be due to perceptual and criterial noise. From Table 3 we see that these percentages range from 7% (for Subject 3 in the size constant condition) to 21% (for Subject 1 in the size constant condition). In Experiment 1 subjects were not told to use a deterministic rule. If they did use a deterministic rule, then the percentage of responses not accounted for by the best-fitting (deterministic) bound should be the same as in Experiment 2. If the Experiment 1 subjects used probabilistic rules, the percentages should be larger than in Experiment 2.

Table 3 indicates that for 4 of the 6 subjects in Experiment 1, the percentage of responses not accounted for by the best-fitting bound ranged from 12% to 18%. These values are in the same range as in Experiment 2 and so we believe the evidence indicates that, except for a few occasional guesses, each of these subjects used a deterministic decision rule. Two of the Experiment 1 subjects (Subject 3 in each condition) apparently guessed more frequently, perhaps in a sophisticated fashion (Broadbent, 1967), as predicted, say, by Nosofsky's (1986) generalized context model. This was especially true for Subject 3 in the orientation constant condition. In this case, not only is the best-fitting bound far from the optimal but it also is associated with a very large SSE and accounts for only 71% of the subject's responses. The hypothesis that Subject 3 frequently guessed is also supported by a low accuracy rate (only 62%).

**Experiment 3**

Experiment 3 was designed to test whether subjects could decisionally integrate size and orientation information, and if they could, how optimally they would do so. It is sometimes assumed to be difficult or impossible to integrate information from separable dimensions, especially under speeded conditions. For example, Grau and Nelson (1988) state that "selective attention may be mandatory with some separable dimensions" (p. 348). Evidence is available, however, that subjects can integrate information from separable dimensions (e.g., Miller, 1982; Nosofsky, 1985, 1986, 1989) even in speeded tasks (e.g., Biederman & Checkosky, 1970; Pomerantz & Garner, 1973) and even when the dimensions come from different modalities (e.g., Melara, 1989; Melara & O'Brien, 1987). Little is known, however, about the optimality of this integration process.

In addition, the apparently inaccurate assumption that subjects are constrained to decisionally separate stimulus components that are perceptually separable underlies a popular test of separability; namely, that no redundancy gain occurs in a speeded classification task with stimuli constructed from separable components when the dimensions are correlated (e.g., Dykes, 1979; Garner, 1974; Garner & Felfoldy, 1970; Lockhead, 1966; Lockhead & King, 1977). The idea is that separable dimensions are processed by separate channels; thus when instructed to categorize on the basis of one relevant dimension, the subject attends to the relevant channel and ignores the one that is irrelevant. In this case, no redundancy gain occurs. A subject responding optimally in this task, however, will integrate information and show a redundancy gain regardless of whether the components are perceptually separable or integral (Ashby & Maddox, 1990; Ashby & Townsend, 1986). Therefore, if information from separable dimensions were easily integrated, this test would not be valid, and so one who uses the test is implicitly assuming that it is impossible or at least very difficult to integrate information from separable dimensions.

The contours of equal probability describing the Experiment 3 categories are shown in Figure 4b and the numerical values of the population parameters are given in Table 1. The prototypes were chosen to have the same values on the size and the orientation dimensions as the Experiment 1 prototypes (note, however, that they are paired differently). Consequently, optimal performance is 80% correct in Experiment 3. All subjects participated in two experimental sessions except for Subject 5, who completed three sessions.

Note that the optimal bound is the line \( y = x \). Ashby and Gott (1988) found that when the stimulus dimensions were horizontal and vertical line length, subjects quickly and spontaneously adopted this bound under these stimulus conditions and applied it very consistently. With line lengths however, the rule \( y = x \) or equivalently \( y - x = 0 \) has an especially simple interpretation. The subject compares the two lengths, gives one response if the vertical line is longer and the other response if the horizontal line is longer. With stimulus dimensions of size and orientation however, this rule translates to something like "compare the size and orientation of the stimulus; if the size is greater than the orientation, give one response, if the size is less than the orientation, give the other." Because the units are different, this is like comparing apples and oranges and so it is not clear that subjects will have as much success in this experiment as they did in the corresponding Ashby and Gott experiment.

**Results and Discussion**

We begin by evaluating the efficacy of the stimulus generation procedure. Table 2 lists the percentage of exemplars accounted for by the optimal bound \( y = x \) during the subjects'
last experimental session. This is the expected percentage correct if the subject had used the optimal rule perfectly. Note that all percentages are within 4% of the 80% population value. Also listed are the linear bounds that best separated the stimulus samples and the percentage of exemplars accounted for by these bounds. With the exception of Subject 1, these all have slopes very close to the optimal value of 1.0 and for all subjects, including Subject 1, the best-fitting bound does not account for more of the stimuli than the optimal bound. Finally, Table 2 lists the percentage of exemplars accounted for by the independent decisions bounds. In all cases, these percentages are more than 5% below the percentages accounted for by the optimal bound. Therefore, the Experiment 3 stimuli were representative of the Figure 4b contours of equal probability.

Figure 6 shows the data of 1 subject (Subject 5) during his or her last experimental session and Table 3 describes the responses of all 5 subjects during their last session. First, note that the accuracy of all subjects was at least 70% and that in each case, this is within 10% of the optimal percent correct. In two cases, accuracy rates were higher than predicted by the most accurate independent decisions rule.

Table 3 also indicates that the linear bound that best separates each subject's responses has a slope considerably less than the optimal value of 1.0. The optimal slope is 1.0 only if the subject equates 250 screen units with an orientation of π/4 radians. One possibility is that each of the subjects equated 250 screen units with an orientation of less than π/4 radians and then assumed (mistakenly) that after using this translation, the optimal rule was y = x. The problem with this interpretation is that it does not explain the large intercept terms associated with the best-fitting linear bounds. In fact, note that for all 5 subjects, the smaller the slope, the larger the intercept. Data from a subject who placed all attention on the orientation dimension and ignored the size dimension would be described by a slope of zero and, if he or she responded A and B about equally often, by a large intercept (i.e., 225). Therefore, we believe that a more plausible hypothesis is that although all subjects integrated information from the two stimulus dimensions, each subject focused more attention on the orientation dimension than was optimal. Although it is not clear why subjects should focus more on orientation than on size in this task, similar results have been found in other studies. For example, Nosofsky (1986) reported data for 2 subjects in a number of categorization conditions that used these same Figure 1 stimuli. In two of these conditions, the optimal rule was to focus equal amounts of attention on the two dimensions. In all four cases, the best-fitting augmented generalized context model indicated that more attention was placed on the orientation dimension. In three of the four cases, the difference in attention was significant at the α = .01 level.

Table 3 also lists the SSE and the percentage of responses accounted for by the best-fitting linear bound and by the optimal bound y = x. Note that in each case, the optimal bound accounts for a large percentage of the responses and that overall, these percentages are higher than in Experiment 1. This suggests that the subjects decisionally integrated the stimulus information and did not respond with a separable strategy. To confirm that subjects integrated the stimulus dimensions, we performed the statistical test of the null hypothesis of independent decisions that was developed by Ashby and Gott (1988). A subject using an independent decisions rule—that is, a subject who treats the stimulus dimensions as separable—divides the x, y stimulus plane into four quadrants (by placing one bound at x = xh and another bound at y = yh). The critical quadrants are the lower left and the upper right. A sample falling in either of these regions contains contradictory information. In the upper right quadrant, both components have large values and in the lower left quadrant, both have small values. Independent decisions models predict that subjects will guess to any exemplar falling in either of these quadrants and therefore that in both of these quadrants the subject's response will not depend on which side of the bound y = x the stimulus happens to fall. Let P equal the probability that the subject responds optimally in these two quadrants (i.e., A to any exemplar falling above the y = x bound and B to any exemplar falling below); then, assuming the null hypothesis that subjects used an independent decisions rule, P = .5. Table 3 contains estimates of P for each of the 5 subjects. For Subjects 2–5, the null hypothesis that P = .5 can be rejected in favor of the alternative that P > .5 with α = .001, and for Subject 1 it can be rejected with α = .005.

One weakness of this test is that it assumes no perceptual or criterial noise. Suppose a subject uses an independent decisions strategy perfectly, with the exception that perceptual noise causes the perceptual coordinates to differ somewhat from the stimulus coordinates and criterial noise causes trial-by-trial variability in the criteria xh and yh. Now consider a trial in which the stimulus falls in the upper right quadrant but in which it is much closer to the vertical than to the horizontal bound. Note that the optimal response in this case is A. Because of perceptual and criterial noise, this stimulus is much more likely to be perceived to fall in the upper left
information they needed to respond optimally as soon as they were shown the prototypes (i.e., before the first practice trial). In Experiment 4 the prototypes suggest that separability is optimal when in fact, because size and orientation are correlated in each category, integration is optimal. All subjects in Experiment 4 completed two experimental sessions, except for Subject 3, who completed only one session.

The contours of equal probability corresponding to the two stimulus categories are shown in Figure 4c, and Table 1 lists the values of the category distribution parameters. Note that the category prototypes (i.e., the means) have different sizes but the same orientations. Therefore a subject who categorizes on the basis of similarity to prototype will adopt a bound parallel to the orientation axis. Such a subject is treating the stimulus dimensions as dimensionally separable.

However, Table 1 also indicates a large positive correlation between size and orientation within each category. Because of this correlation, the optimal strategy is not to treat the dimensions as separable but to integrate information from the two dimensions with the rule $y = x$ (Ashby & Gott, 1988). To understand this, note that an exemplar described by the coordinates $(x_0, y_0)$ in Figure 4c is closer to the A prototype but is more likely to be a member of Category B. If subjects integrate size and orientation in this task, then it is difficult to see how decisional separability can be considered to be a property associated with these dimensions.

Results and Discussion

As before, we begin by evaluating the efficacy of the stimulus generation procedure. Table 2 lists the percentage of exemplars accounted for by the optimal $y = x$ bound during each subject’s last experimental session. The category parameters were chosen so that the optimal responder would be

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This possibility was suggested by Rob Nosofsky (personal communication, March 16, 1989).

We begin with $\alpha = 20$ because the independent decisions model predicts an increase in these proportions from $\alpha = 0$ to $\alpha = 20$ but virtually no change for larger $\alpha$. The rationale is as follows. Suppose the independent decisions model is correct. Then, for example, an exemplar falling in the upper left quadrant might be perceived as falling in the lower right and so elicits a B response. However, perceptual noise is extremely unlikely to cause a change in perceived value of more than 20 units. Therefore for $\alpha > 20$, this effect should be minimal.

---

Table 4

<table>
<thead>
<tr>
<th>$\alpha$</th>
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<th>3</th>
<th>4</th>
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<td>.836</td>
<td>.982</td>
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</tr>
<tr>
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<td>.957</td>
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<td>1.000</td>
<td>.985</td>
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</table>

---

Experiment 4

We designed Experiment 4 to assess the limits on the ability of the decision process to integrate information from the size and orientation dimensions. One advantage of the randomization technique is that we can investigate the optimality of categorization. There are many ways to integrate stimulus information, but only one of them is optimal (in the sense of maximizing accuracy). In Experiment 3 the optimal rule was minimum distance classification, and so subjects had all the
correct with .80 probability. Also listed in Table 2 are the percentage of exemplars accounted for by the most accurate separable strategy. Note that in each case, the optimal rule predicts accuracy to be at least 11% higher. Finally, Table 2 lists the linear bound that best separates the exemplars from the two categories along with the SSE and the percentage of stimuli correctly categorized by this bound. Note that these best-fitting bounds are very close to the optimal bound \( y = x \).

Figure 7 illustrates the responses for Subject 3 during his or her last experimental session and Table 3 describes each subject’s performance during his or her last experimental session. In each case, the percentage of correct responses is well below the optimal, confirming the difficulty of this task. However, note that Subjects 1 and 3 each outperformed the most accurate separable decision strategy (i.e., the minimum distance classifier). Subject 1 performed 3 percentage points better, and Subject 3 performed 7 points better. This suggests that these subjects may have decisionally integrated the component information.

Table 3 also lists the equations of the linear bounds that best separated the subjects’ responses. Note that in each case, the best-fitting bound is close to the optimal \( y = x \) bound. Finally, Table 3 lists the SSE and the percentage of responses accounted for by (a) the best-fitting linear bound, (b) the optimal \( y = x \) bound, and (c) the minimum distance bound \( (x = 250) \), which in this case assumes decisional separability. Note that for each subject the SSE for the \( x = 250 \) bound is much larger than the SSE for the optimal bound and, except for Subject 2, both the best-fitting bound and the optimal bound account for a larger percentage of responses than the minimum distance bound. Even so, the best-fitting bound mispredicts many responses in each case. This indicates that subjects found this task to be extremely difficult and that they resorted to guessing (or probabilistic responding) on a signifi-

![Figure 7. Response data for Subject 3 in Experiment 4. (A square indicates an A response, a cross a B response.)](image-url)
some cases than in others. For example, if the outputs of the perceptual channels are in different units (as with size and orientation), decision rules that involve information integration may be difficult to learn. Similarly, it may be difficult to decisionally separate dimensions that are perceptually integral.

These results have important implications for several currently popular methods of testing for perceptual separability. For example, a redundancy gain in speeded classification is interpreted as evidence of perceptual integrality (e.g., Dykes, 1979; Garner, 1974; Garner & Felfoldy, 1970; Lockhead, 1966; Lockhead & King, 1977). Unfortunately, the optimal decision rule in this task is to integrate the component information and therefore, even if the components are perceptually separable, a redundancy gain should be expected from experienced, motivated subjects (Ashby & Maddox, 1990; Ashby & Townsend, 1986). Testing for a redundancy gain in speeded classification is not a good method for identifying perceptual separability.

Another example occurs in restricted classification tasks in which subjects are presented with a set of stimuli and are asked to indicate which stimuli belong together (e.g., Burns et al., 1978; Grau & Nelson, 1988; Handel & Imai, 1972; Slep, Burns, & McDonough, 1980). The stimulus dimensions are assumed to be perceptually separable if a bound that is parallel to one of the stimulus coordinate axes separates the subject's categorization responses; that is, if decisional separability holds. If decisional separability fails, the dimensions are assumed to be perceived in an integral fashion. This paradigm tests explicitly for decisional separability. Inferences can therefore be made about perceptual separability, only if some empirical correlation exists between the two. Our results indicate that if this correlation exists, it is imperfect, and so we do not believe that restricted classification is a good paradigm for studying perceptual separability.

The classification task in which dimensional values vary orthogonally is a much better technique for testing perceptual separability because decisional separability is likely to be the optimal decision rule in this task (Ashby & Maddox, 1990). Other powerful tests of perceptual separability were suggested by Ashby and Townsend (1986) and by Ashby (1988). More specifically, however, we hope that carefully distinguishing between perceptual and decisional forms of separability will lead to an increased understanding of the interactions that can occur when several stimulus dimensions are processed simultaneously.

References

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