Journal of Experimental Psychology:
Human Perception and Performance
1989, Vol. 15, No. 2, 384–393

0096-1523/89/$00.75

OBSERVATIONS

Understanding Natural Dynamics

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When making dynamical judgments, people can make effective use of only one salient dimension of information present in the event. People do not make dynamical judgments by deriving multidimensional quantities. The adequacy of dynamical judgments, therefore, depends on the degree of dimensionality that is both inherent in the physics of the event and presumed to be present by the observer. There are two classes of physical motion contexts in which objects may appear. In the simplest class, there exists only one dynamically relevant object parameter: the position over time of the object's center of mass. In the other class of motion contexts, there are additional object attributes, such as mass distribution and orientation, that are of dynamical relevance. In the former class, objects may be formally treated as extensionless point particles, whereas in the latter class some aspect of the object's extension in space is coupled into its motion. A survey of commonsense understandings showed that people are relatively accurate when specific dynamical judgments can be accurately based on a single information dimension; however, erroneous judgments are pervasive when simple motion contexts are misconstrued as being multidimensional, and when multidimensional quantities are the necessary basis for accurate judgments.

In this article we present an account of dynamical event complexity and a variety of research findings that support its predictions. In essence, we propose that people make judgments about natural object motions on the basis of only one parameter of information that is salient in the event. By this account, people encounter difficulties when construing dynamical events that are inherently multidimensional, or that have been incorrectly defined by them as being multidimensional.

Our account of dynamical event complexity begins with a distinction taken from physics: The dynamically relevant properties of objects are defined by the motion contexts in which they are found. Many contexts, particle motions, can be dynamically analyzed by treating the object as a particle located at its center of mass. Free-fall is a good example of a particle motion; an object's shape, orientation, size, and so forth are all irrelevant to its dynamical behavior in free-fall (assuming a vacuum). Other contexts, extended body motions, require that the object be treated as a multidimensional entity. The rolling of a wheel is a good example of an extended body motion. A wheel's moment of inertia (mass distribution) affects its rolling behavior; thus, it cannot be dynamically treated as a particle. It is important to emphasize that this categorization depends not on whether objects are particulate or extended, but rather on the motion context in which they are encountered.

We propose that people base their commonsense dynamical judgments on one information dimension within an event. People do not make dynamical judgments by deriving multidimensional quantities. This proposal predicts that people will generally make accurate dynamical judgments in (a) one-dimensional (particle motion) contexts, or (b) multidimensional (extended body motion) contexts in which circumstances are such that specific judgments can be accurately based on a single information dimension. People are predicted to make erroneous judgments in (a) one-dimensional contexts that are misconstrued as being multidimensional, and (b) multidimensional contexts in which multidimensional quantities are the necessary basis for accurate judgments.

This article is divided into two parts. In the first we develop our account of dynamical event complexity, and in the second we present a variety of research findings that support its predictions.

This research was supported by Air Force Grant AFOSR-87-0238, National Aeronautics and Space Administration Grant NCA2-255, and National Institute of Child Health and Human Development (NICHD) Grant HD-16195 to Dennis R. Proffitt and by NICHD Postdoctoral Fellowship HD-07036 to David L. Gilden. Mary Kaiser, Roger Shepard, and James Todd provided valuable criticism on earlier versions of this article.

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Complexity in Natural Motions

Successful appreciations of natural motions depend on the kind of object that is being viewed and the dynamical context in which the viewing takes place. Psychological theories of commonsense understandings of natural dynamics must ultimately refer to classical mechanics, because it is in this field that the dynamics of object motions are articulated. The following treatment of analytical dynamics introduces those notions that provide the basis for assessing human abilities in understanding natural motions.

We want to make two basic ideas explicit in this section. The first is that from a formal point of view, the laws of object motion are independent of both the object and the motion under consideration. All equations of motion are derived from a single minimum principle. The second idea is that there is a hierarchy of object complexity that is manifested when the symmetries in dynamical systems are analyzed from the point of view of invariances in the equations of motion. This hierarchy is especially interesting because there exists a definite limit to object simplicity, and, as will be discussed in later sections of this article, this limit is reflected in human performance: Human competence at understanding dynamical systems approaches adequacy only for those systems in which object simplicity is maximum.

The Formal Unity of Natural Motions

Classical mechanics has a historical primacy in physics due to the perceptibility of relevant information. In the other three major branches of physics—electrodynamics, thermodynamics, and quantum mechanics—the individual motions of the relevant particles are invisible. Mechanics, however, treats the motion of rigid bodies that can be seen. Mechanical systems were the first studied because the objects that constitute them were the first noticed.

The familiarity that we have with mechanical systems is, however, highly deceptive. There is a coherence in mechanical systems that is revealed only when attention is withdrawn from how these systems appear phenomenally and an abstract point of view is taken. This point of view begins with the replacement of the three-dimensional space of world experience with an appropriate mathematical space that more adequately describes the environment of the mechanical system. This idea is developed more fully and generally in the Appendix. Here we will take a somewhat more elementary and specific point of view, and discuss the mathematical transformation that relates linear momentum and angular momentum systems.

One of the key themes of this article is that people are relatively competent in making judgments about systems governed by linear momentum conservation, but that they are much poorer in judging systems where angular momentum is relevant. Mathematically, these two systems are isomorphic, in that there is a single operation that carries the linear domain into the angular domain. This operation is the cross product with the position vector. In this way forces are mapped into torques, linear momentum is mapped into angular momentum, and Newton's third law, $F = ma$, is mapped into $\tau = dL/dt$. This mapping is in fact the manner in which angular systems are introduced pedagogically. Physics textbooks begin with a discussion of linear momentum systems. Not only are these systems simpler in terms of their mathematics, but, as we will argue, people have fairly good intuitions about their behavior. In a subsequent section of physics textbooks, the cross product is introduced, and the equations for angular momentum systems are derived. At this point, students are introduced to a set of amazing demonstrations that capitalize on precession and the orthogonality of torque and angular momentum to perceived object motions. The difficulty that students have with these concepts is discussed below. The learning of physics requires that students understand the nature of the isomorphism that relates linear and angular systems, and it is, in fact, this isomorphism to which experienced physicists return when asked to explain the unusual behavior of angular systems. They will simply state that $F = ma$.

A Duality in Motion Contexts

The unity and elegance that characterize a mathematical description of natural motions do not characterize commonsense understandings. Here we introduce the notion that there is a hierarchy of dynamical event complexity, and that human understanding is most competent with those systems at the bottom of the hierarchy. There are three concepts that are critical in the determination of this complexity hierarchy. The first is that complexity is defined in terms of the motion context in which objects appear—not in terms of the objects themselves. The second idea is that the complexity of a mechanical system is related to the symmetries that it possesses. The systems with the greatest degree of symmetry are the least complex. Finally, there is a special class of mechanical systems in which symmetry is maximal; such systems treat the objects within them as extensionless point particles. In all other mechanical systems there is some aspect of the extension of the object in space that is relevant for its motion. What it means for an object to appear in a motion context, and the sense in which a mechanical system has symmetries, are the subjects of this section.

A mechanical system is a collection of objects moving under the action of external and internal force fields. The properties of individual objects that are dynamically relevant are determined by the motions that they are executing. In this sense, a mechanical system is a context for the objects within it. This notion is best illustrated by a simple example.

Consider the two following contexts for the motion of a top: (1) free-fall of a top that has been dropped in a gravitational field and (2) precession of a spinning top that is balanced on a pedestal in a gravitational field. Both are examples of a top falling, but the two motions are quite different; the properties of the top that are of dynamical relevance also differ depending on the context. For example, the shape of the top matters only if a torque is applied to it. The trajectory of the center of mass of a spinning top in free-fall is identical to that of a nonspinning one. For any object in free-fall in a uniform gravitational field, the integrated torque, computed about the center of mass, is zero. On the other hand, a top that is supported by a pedestal is subject to a gravitational torque about the point of contact. In this situation, spinning is relevant to the nature of the top's fall. A nonspinning top falls down; a spinning top falls sideways—that is, it precesses. Spinning tops have many more dynamically relevant features. The basic point here is that it is not the object per se that determines its motion; it is the motion, or more precisely the mechanical system, that characterizes the object. The complexity of objects is a reflection of the complexity of the mechanical systems in which they appear.

The complexity of a dynamical system is determined by its symmetries. Symmetry in a dynamical system is related to the more familiar notions that people have of figural symmetry, but it is not quite the same thing. Figural symmetries are defined in terms of invariance under a class of transformations that include translation, rotation, and reflection. Figures with a high degree of symmetry will be invariant under several of these transformations. Symmetry in dynamical systems is similarly defined, except that the object that undergoes the transformation is a mathematical equation (one of the equations of motion), and the transformation can be quite general. The transformations are generated by changing object attributes, and the result of a transformation is determined by the resulting form of the equations of motion.

An important event, of particular interest in this article, is the motion of a wheel rolling down an inclined plane. We present here the physics of the rolling wheel to illustrate the concept of dynamical symmetry. In Figure 1, we illustrate two rimlike wheels. One is perched on an inclined plane; the other is held by a thread that will be cut. These two situations define two mechanical systems. Conservation of energy for these systems is written

$$\frac{1}{2}Mv^2 + \frac{1}{2}Iw^2 = Mg(h_0 - h): \quad \text{rolling} \tag{1}$$

$$\frac{1}{2}Mv^2 = Mg(h_0 - h): \quad \text{free-fall,} \tag{2}$$

where M is the mass of the wheel, B is its inner radius, A is its outer radius, v is its instantaneous velocity, w is its angular velocity about its center of mass (C_m), and the moment of intertia is written

$$I = \frac{1}{2}M(A^2 + B^2). \tag{3}$$

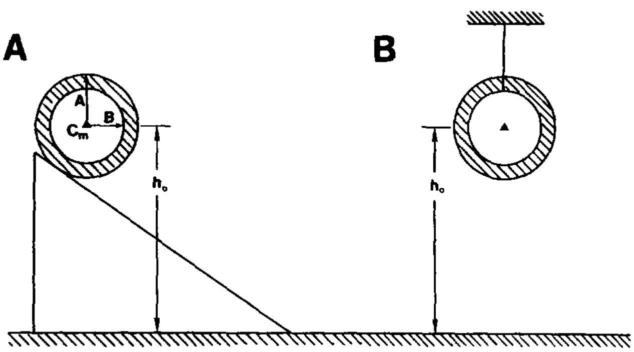

Figure 1. Two motions contexts for a rimlike wheel. In Panel A the wheel is perched on an inclined plane. In this context the mass distribution, given by the ratio B/A, influences the rate at which the wheel rolls. Mass and the overall size of the wheel are not relevant. In Panel B a wheel is attached to a thread that will be cut, resulting in free-fall. In this context there is no attribute of the wheel that affects its velocity. A free-fall context regards all objects as being extensionless point particles located at their center of mass (C_m).

We suppose here that the wheel rolls without slipping, so that its velocity down the ramp can be written $v = Aw$. Solving for the instantaneous velocity of the wheel as a function of the vertical height yields

$$v = \left[\frac{2g(h_0 - h)}{3/2 + 1/2B^2/A^2}\right]^{1/2}: \quad \text{rolling} \tag{4}$$

$$v = [2g(h_0 - h)]^{1/2}: \quad \text{free-fall.} \tag{5}$$

Analysis of these equations reveals their symmetries. These two systems are essentially distinguished by the fact that, for the rolling wheel, kinetic energy is partitioned into both a translational part and a rotational part, whereas for the falling wheel, all kinetic energy is translational. We have canceled out mass from both of these equations, indicating that the mass of the wheel does not influence its motion. This is a symmetry that the systems share. When the mass term is canceled, there is nothing left in the equation for the falling wheel that tells that a wheel is being described. A falling wheel can be distorted in any manner, and it will fall along the same trajectory; free-fall is a motion context in which objects are treated as extensionless point particles. The rolling wheel, however, does not possess this symmetry. The ratio B/A is present; it defines how mass is distributed in the rim. Note that any transformation of the rim that leaves the ratio B/A invariant will have no effect on the motion. Thus, the rolling wheel is invariant under an overall size transformation, $A \rightarrow pA$, $B \rightarrow pB$, but it is not invariant under a fattening or thinning transformation, $A \rightarrow pA$, $B \rightarrow B$. The existence of a transformation on the spatial properties of the rolling wheel, for which the equation of motion is not invariant, is crucial; the rolling wheel is not being treated as an extensionless point particle—its extension in space is reflected in its motion.

To summarize, objects may appear in mechanical systems in two distinct ways. The distinction is defined by the symmetries of the mechanical system. If a mechanical system is invariant under all transformations that operate on the three-dimensional shape and orientation of the objects in motion, then those objects are being treated by the system as extensionless points. Such objects are referred to as *point particles.* The point particle is characterized only by its position in space and is the simplest object that can exist in a mechanical system. All other objects are treated by their systems as being extended. Examples of extended body systems are ones in which the objects are subjected to torques, are floating, or are moving through a resistive medium.[1]

[1] The notion of dynamical symmetry is related to that of generalization in understanding. The more object properties that cancel out of the equations of motion, or alternatively the higher the degree of symmetry in the mechanical system, the more easily any given encounter with the system will generalize. For example, one has only to see a single example of a relatively heavy object fall to understand free-fall. All objects not affected by environmental circumstances, such as wind, free-fall along parabolic trajectories. On the other hand, every top behaves differently depending on the details of its construction and on the precise manner in which it is launched. Those systems that have maximum generality are those that treat their objects as point particles.

Extended Bodies and Multidimensionality

The symmetry of a mechanical system is reflected in the amount of information required to represent its dynamics. Those systems in which symmetry is maximal, point particle systems, have exactly one relevant category of information. It is in this sense that particle motions are one-dimensional and are treated in dynamics as a special case.

The dimensionality of an object in a mechanical system is determined by the number of object attributes that can influence its motion. Point particle systems contain particle position as their single category of information. The very act of looking at the event being displayed is simultaneous with noticing the single dimension required for dynamical analysis of the event.

The vast majority of mechanical systems define extended body motions. Extended body motions are inherently multidimensional, in the sense that there is some spatial property of the object, apart from where it is located, that is coupled into its motion. The distinction between point particles and extended bodies is essentially between motion contexts that couple into only particle location and contexts that couple into additional spatial attributes of the body.

The first and most important step in the analysis of multidimensional systems is the formation of multidimensional quantities. Such quantities are formed through some sort of multiplication; it does not make sense to add quantities that have different units (dimensions). The kind of multiplication that is appropriate depends on the quantities being combined. For example, torque is formed by the cross product between position and force. Construction of the moment of inertia requires an integration over the mass distribution weighted by the squared distance. Unlike position, such multidimensional quantities are not categories of perception.

In Table 1, we list five extended body mechanical systems that we have studied in terms of commonsense understandings. For each system we show (a) the different dimensions of information that are of dynamical relevance, (b) the appropriate physical representation for the system, and (c) the operations by which dynamically relevant multidimensional quantities are formed.

Commonsense Understandings of Dynamics

We believe that people do not derive multidimensional quantities when observing natural motions. Thus, we predict that their accuracy in making dynamical judgments will be related to event dimensionality. In particular, we predict that people will exhibit pervasive failures when construing extended body motions. The following section summarizes observations and research that support this prediction.

People's Dynamical Understandings of Extended Body Motions Are Relatively Poor

As previously discussed, dynamical analyses of extended body motions are far more complex than are analyses of simple particle motions. A particle motion manifests only one dimension of dynamical relevance—the position over time of its object's center of mass. This single category of information can be placed directly into all dynamically relevant representations of the event. On the other hand, an extended body motion, by definition, manifests multiple categories of dynamical information, and thus introduces additional processing requirements. For such events, multiple parameters of information must be noticed and combined through multiplicative operations; only after performing these operations can the resulting multidimensional quantities be composed within dynamical representations.

Students in introductory physics courses encounter numerous difficulties when they move from treatments of particle motions to extended body dynamics. We asked 19 high-school physics teachers to provide an ordered list of the three concepts that their students find most difficult to understand (Proffitt, Kaiser, & Whelan, 1989). The most frequently mentioned concept was angular momentum. It was mentioned by 18 of the teachers and was listed first by 10 of them, as compared with 4 first-place listings for the next most frequently mentioned concept. The difficulty of teaching extended body dynamics came up again in follow-up discussions with these teachers, interviews with University of Virginia physics professors, and a personal communication with Jearl Walker, author of "The Amateur Scientist" in *Scientific American*.

Typically, introductory physics courses start with a review of measurement issues, and then move to treatments of point particle kinematics (pure motion considerations for particle systems). With this background established, dynamics is introduced, beginning with Newton's laws, and the treatment is restricted to point particle systems. The kinematics and dynamics of extended body motions are addressed next, and it is at this juncture in the course that many students begin to experience profound difficulties, and for some a catastrophe, in understanding the material. In learning the principles of extended body motions, students are exposed to a more purely mathematical treatment of the topic, and are required to reason abstractly about motions that cannot be perceptually appreciated or imagined. Typically, what was learned about particle motions could be reconciled with commonsense views about the workings of the world. On the other hand, the dynamics of extended body motions defy assimilation into commonsense reasoning.

The following is an initial assessment of people's competence in dealing with dynamics in a sample of extended body motions. Although this work is still quite preliminary, it clearly reveals a dearth of dynamical understandings.

Top and gyroscopic motions. The best example of perception's failure to penetrate the dynamics of complex extended body motions can be experienced by watching a top or gyroscope. Tops and gyroscopes are enduring toys because their wonder-producing motions are not penetrated by perceptual experience.

The Nobel laureate, Feynman (1975) wrote in his lecture series to the freshmen at California Institute of Technology, "The precession of a top looks like some kind of *miracle* [italics added] involving right angles and circles, and twists and right-hand screws" (pp. 20–26). In a similar vein, Walker (1985) wrote,

> Rotation is fascinating because in spite of its common occurrence it is difficult to understand . . . . For example, when I spin a top I am always amazed that it stands upright in spite of the gravity

Table 1
Five Extended Body Mechanical Systems

System	Information	Representation	Computation
Tops and gyroscopes	Mass distribution, angular velocity, torque	Algebra of cross products, motion in effective potential	Cross products: $L = r \times v$ $\tau = r \times g$
Rolling wheels	Mass distribution, rotation/translation	Energy conservation	Energy partition, moment of inertia
Balances	Weight and distance from fulcrum	Leverage	Multiplication (cross product)
Collisions	Speed and angle	Energy/momentum conservation	Multiplication (dot product)
Displacements	Floating/sinking, mass, volume	Archimedes' principle	Boolean algebra

tugging at it. Even when it leans over, it does not fall. Instead it precesses, that is, the central axis about which it spins circles around the vertical. *Such motion is magic* [italics added]. (Preface)

In the above quotation, Walker (1985) pointed out an important aspect of people's failure to comprehend the apparent gravity-defying behavior of gyroscopes: Rotating objects are common in everyday experience. It is not the case that tops and gyroscopes are difficult to understand simply because people have limited experience with their motions.

Proffitt, Kaiser, and Whelan (1989) interviewed 50 undergraduates about their understanding of bicycles and gyroscopes. With respect to bicycles, one of the questions asked why a rapidly moving bicycle was easier to balance than one that was barely moving. Not one subject mentioned that it might have something to do with the rotation of the bicycle's wheels. In fact, bicycle wheels act as gyroscopes and contribute to stability in proportion to their angular velocity. (Although there is controversy in the physics community about what factors influence bicycle stability, it is agreed that the gyroscopic properties of bicycle wheels have a large influence [Jones, 1970; Kirshner, 1980; Lowell & McKell, 1982].) Most subjects gave erroneous reasons that related to linear momentum (e.g., because the bicycle is moving forward, it is harder to tip over) or to such invisible factors as the wind (e.g., the faster one goes, the more the wind builds up on the sides, forming a resistance to falling over).

These subjects were also asked to predict the behavior of a spinning gyroscope. Most subjects predicted that it would fall off its pedestal at an orientation in which it would, in fact, balance. The subjects then viewed a spinning gyroscope; however, this experience afforded only amazement, as none of the subjects could provide a reasonable account of its apparent gravity-defying behavior. Commonly given answers to the question about what prevents a gyroscope from falling were "the wind," "the spinning wheel acts as a propeller," or "the wheel is spinning." Never was it correctly mentioned that the precession of the gyroscope is the natural result of gravity. In fact, precession is a relatively slow motion compared with the spinning of the wheel, and it seemed to have been essentially ignored by these subjects.

Proffitt, Kaiser, and Whelan (1989) also conducted similar interviews with 25 members of the University of Virginia Bicycling Club. These bicycle racers rode their bicycles about 100 mi (161 km) per week on average. Interviews in greater depth with these individuals revealed that their understanding of what accounts for the stability of a moving bicycle differed little from that of those subjects with far less familiarity with bicycles. In particular, only 2 of the cyclists predicted that if they leaned while riding their bicycle, then the bicycle's front wheel would turn in the direction of the lean without their having to turn the handle bars. (This turning of the bicycle's wheel is an example of precession.) Moreover, these cyclists did not show an appreciably better understanding of the gyroscope's motions.

As is witnessed in the above quotations from eminent physicists, and in our own interviews with university and high-school physics teachers, gyroscopic motions are understood by physicists in only a formal mathematical way, and are not appreciated by them at a perceptual level. Physicists attest to their enjoyment of the seemingly magical behavior of tops and gyroscopes because their formal appreciation of the dynamics of these objects is not grounded in their phenomenal acquaintance with these motions.

Rolling wheels. Recent experiments have made it increasingly evident that when confronted with problems in which the rotation of extended bodies gives the relevant physical description, people very frequently fail to understand the system's dynamics (Proffitt, Kaiser, & Whelan, 1989). Fifty subjects were shown and allowed to handle wheels that varied in mass, radius, or mass distribution (solid wheels vs. rims). On each trial, two wheels were paired with each other such that two of the variables were held constant (e.g., mass and mass distribution) and the other attribute varied (in this case, radius). The wheels were placed at the top of an inclined plane, and subjects were asked to predict whether both wheels would arrive at the bottom at the same time or at different times, and, if at different times, which wheel would reach the bottom first. The results were as follows: (a) When mass and radius were varied, subjects chose as correct each of the three alternatives in about equal numbers. (Only about a third of the subjects correctly stated that the wheels would reach the bottom at the same time.) (b) When mass distribution was varied, more than 80% of the subjects incorrectly predicted that the wheels would roll down at the same rate. As discussed in the earlier section on natural motions, the only relevant variable in this experimental situation is mass distribution; the more the mass is concentrated toward the rim, the slower the wheel will roll. In the only case in which the subjects had a strong systematic bias as a group, then, that bias was incorrect. Releasing the wheels and allowing the subjects to observe the outcomes caused the subjects to become perplexed, and did not enlighten them about why the wheels rolled at the rates that they did.

This experiment was run with experts in angular systems—University of Virginia Department of Physics faculty members—and with people who are especially familiar with rolling wheels—25 members of the University of Virginia Bicycling Club. When the former group was prohibited from solving the problems analytically, neither group did appreciably better than the naive undergraduates.

In another study, undergraduates were assessed for their perceptual appreciation of the influence of mass distribution on rotational dynamics (Kaiser, Grunwald, & Proffitt, 1987). The observers were shown a simulation of a satellite spinning in space. The satellite could undergo changes in its mass distribution by the opening or closing of panels. The observers' task was to judge whether the resulting change in angular velocity was natural or could have been caused only by the application of some external, unseen force. When the satellite's panels opened, the natural event showed the angular velocity decreasing. Anomalous events showed the satellite's angular velocity (a) changing direction, (b) stopping altogether, (c) slowing down too much, (d) remaining unchanged, and (e) speeding up. It was found that observers judged the events in which the satellite stopped or changed direction as appearing anomalous, but judged all the other events as looking equally natural. The observers, that is, showed a qualitative appreciation for the preservation of angular velocity direction, but no sensitivity to the principles governing the conservation of angular momentum—specifically, the influence of changing mass distribution on angular velocity. This result converges with the results of the previous study that showed observers to be insensitive to the influence of mass distribution on the rate at which wheels rolled down an incline.

Balances. In a preliminary study, we gave 180 undergraduates five paper-and-pencil balance problems. In each of these problems, some blocks were depicted as placed on the left arm of a two-arm balance. In addition, there was a set of blocks depicted to the right of the balance. The task was to indicate where the blocks should be placed on the right arm to balance with the blocks on the left. In the instructions that accompanied the problems, there was an explicit warning that there might be no place where the blocks could be placed on the right arm for the given configuration of blocks on the left arm. Subjects were instructed to mark such a configuration with a question mark. There was, in fact, one impossible problem included in the quiz. The results were that 29% made no mistakes, 28% missed one, 20% missed two, 11% missed three, 9% missed four, and 3% missed all five problems. About 75% of those that missed only one erred on the impossible problem. If this problem had been omitted, about half of the sample would have scored perfectly.

It is evident that about half of our subjects did not understand fully the principle of the balance, and that at least a quarter of them were severely incompetent in this domain. These results are consistent with those found by Siegler (1978).

Collisions. When a moving object strikes a stationary one (imagine billiard balls with unequal masses), the motions of the objects provide sufficient information to specify the unique relative masses of the objects involved. In particular, the projection of postcollision velocities onto the collision axis forms a ratio equivalent to the objects' mass ratio. The relevant information for mass ratio judgments, then, is the multidimensional quantity of projected velocity.

Gilden and Proffitt (1989) required observers to make mass ratio judgments while viewing computer simulations of natural collisions. They found that people based their judgments not on projected velocity but rather on one or the other of two information parameters. Judgments were based either on velocity (fast-moving balls were judged to be lighter) or on ricochet (the ball that bounced backward was judged to be lighter). In many situations this heuristical reasoning resulted in accurate judgments; however, when these two parameters were placed in conflict, performance for the group dropped to almost the level of chance performance.

We take this result to be a general finding: Dynamical intuitions about extended body motions are good only when specific judgments can be accurately based on a single information dimension.

Volume displacements. The structure of extended forms obviously comes into consideration when one is making judgments about volume displacement. Understanding displacement problems requires that displacing objects be construed in terms of the parameters of mass and volume.

There is a famous anecdote in which Robert Oppenheimer (the leader of the Manhattan project), Felix Bloch (1952, nobel laureate), and George Gamow (a celebrated quantum theorist) were given a seemingly simple displacement problem: Consider a boat with a weight on it floating in a tank of water. The water level is marked on the tank. If the weight is taken off the boat and placed in the water, will the water level be higher, lower, or the same as the original mark? All three physicists answered incorrectly that the water level remains unchanged (Walker, 1977). The correct answer is that the water level goes down, because when the weight is in the boat it is displacing the volume of water equal to its mass, and when it is in the water it displaces only its own volume.

Whelan (1987) asked undergraduate subjects displacement questions of two sorts: simple questions that could be answered correctly by referring to one parameter of the displacing object and complex questions that required that two parameters of the displacing object be noticed. The following is an example of a simple question: Two objects of equal size but of different weights are placed in identical containers holding the same amount of water. Both objects float. Will the water levels in the two containers remain the same, or will one be higher than the other; and, if one container will have a higher water level, which one will it be? In this question the only relevant parameter of the displacing object is its mass. In questions in which the displacing object is sunken, object volume is the only relevant dimension. The complex questions asked were like the problem mentioned above regarding the weight on the boat. In such questions, the displacing object has two relevant parameters: mass, when the object is floating with the boat, and volume, when the object has sunk. Whelan created a wide variety of simple and complex questions, distinguished by whether the displacement context made a single object parameter relevant or required an appreciation of two object parameters.

It was found that observers gave correct answers 78% of the time for the simple questions and responded correctly only 35% of the time for the complex questions (chance performance being 33% correct for both types of question). Thus, when they considered problems of displacement in contexts in which only one object parameter was relevant, people acted as if they had a good knowledge of Archimedes' principle. When the context made two parameters dynamically relevant, people's performance fell dramatically.

In another experiment, Whelan (1987) had people view an enactment of the weight-on-the-boat problem. An aquarium was modified so that its water level could be raised or lowered quickly and without creating turbulence. Moreover, the observer could not see the mechanism that caused this change. A toy boat carrying a heavy weight was placed in the water, and the observer was instructed to watch as the weight was lifted off the boat and placed into the tank. After being informed that the water level could be manipulated at the moment when the weight went into the water, the observer's task was to judge whether the water achieved its natural level or had been influenced by the experimenter.

It was found that the people were very good at judging when the tank's water had been manipulated by the experimenter. Although most people answered the weight-on-the-boat problem incorrectly when it was asked verbally—most reported that the water level would be the same when the weight was on the boat and when in the water—almost everyone who made this erroneous prediction saw the enactment of the event as highly anomalous; in fact, this contrived event was typically viewed with considerable amusement. The reason for the improved performance in this ongoing, perceptual context was clear: In the weight-on-the-boat problem, the relevant parameters of the displacing object were separated in time during the ongoing event.

The superior performance that was observed when people were presented with an ongoing enactment rather than a verbal presentation of the problem, is a general finding to which we will turn in a later discussion. In some mechanical systems, object dimensionality is segregated in time when the ongoing event is observed. In the above example, observing the weight as it is taken out of the boat allows one to see how this *heavy* object produced a large displacement. Observing the *small* weight being placed into the tank, and watching the water level rush back to its initial level, induces considerable mirth, because the object's size has now become so salient. The dimensions of weight and size are separated in time in the ongoing event but not in the verbally presented problem. Commonsense understandings of Archimedes' principle appear to be good when assessed in simple contexts, but poor when assessed in complex ones.

People Perform Poorly in Particle Motion Contexts That Are Misconstrued as Being Multidimensional

Recently a large number of investigations have been published on people's commonsense beliefs about dynamics (Caramazza, McCloskey, & Green, 1981; Champagne, Klopher, & Anderson, 1980; Clement, 1982; Kaiser, Jonides, & Alexander, 1986; Kaiser, McCloskey, & Proffitt, 1986; Kaiser, Proffitt, & McCloskey, 1985; McCloskey, 1983a, 1983b; McCloskey, Caramazza, & Green, 1980; McCloskey & Kohl, 1983; McCloskey, Washburn, & Felch, 1983). These studies on dynamical understandings, "Intuitive Physics" (McCloskey, 1983a), have been interpreted as showing that people often hold erroneous beliefs about simple object motions. We propose a somewhat different interpretation for this literature.

Although seemingly unintentionally, almost all intuitive physics studies investigated people's understandings of motions in point particle systems. Even though the objects presented in these studies were extended forms, such as balls rolled through C-shaped tubes, bombs dropped from airplanes, and coins tossed in the air, the relevant dynamics in these events are fully specified by the motion of the objects' centers of mass.

We believe that people become muddled on these problems because they misconstrue them as being extended body systems. McCloskey and his colleagues constructed problems that typically presented an extended body system—for example, a pendulum swinging back and forth. Something happens that transforms this system into a particle motion—the pendulum tether breaks—and the participants are asked to predict the ensuing motion. In this problem, the object's dimensionality must be segregated by reasoning across the event's pre- and post-tether-breaking epochs.

Kaiser, Proffitt, and Anderson (1985) and Kaiser and Proffitt (1986) investigated most of McCloskey and his colleagues' (Caramazza et al., 1981; McCloskey, 1983a) situations by having people make judgments first in paper-and-pencil contexts and then when viewing animated computer graphics simulations of these events. The results McCloskey and his colleagues had obtained were replicated for the paper-and-pencil problems; however, it was found that when viewing ongoing displays, people view their erroneous predictions as anomalous, and select natural motions as being correct. As was found in the Archimedes' principle study, animation segregates the dimensionality of these events in time.

Consider the example of the pendulum problem. Most incorrect responses to the paper-and-pencil problem occur when subjects are asked to predict the trajectory that the bob would take if the tether broke at the instant when the bob was at the apex of its arc. Most erroneous responses predict that the bob will fall along a parabolic path rather than straight down. At the instant that the bob is at its apex, it is stationary. Ask anyone what happens when a stationary object is dropped, and they will predict a straight-down trajectory. The difficulty that people have with the pendulum question clearly involves their inability to construe the state of the bob's motion at the instant when the tether breaks. When a person is viewing the ongoing event, the object's extended and particle motion contexts—swinging versus falling—are clearly separated in time.

It should be emphasized here that the advantage that ongoing displays have been shown to have in eliciting accurate dynamical intuitions is restricted to situations in which accurate judgments can be based on single object dimensions. For cases in which impressions of emergent multidimensional quantities must be formed, for example, when a person is evaluating the dynamics of a spinning top, viewing the ongoing event does not spontaneously result in better dynamical intuitions.

Conclusion

In this article we have provided an account of dynamical event complexity, and have related this account to what is known about commonsense dynamical understandings. We find that commonsense dynamical understandings are good only when people can accurately base their judgments on a single dimension of information present in the event.

Our account of dynamical event complexity begins with a recognition that there exists a definite limit to the simplicity of mechanical systems. By placing all object motions that adhere to this limit within one class of object motions, we define two categories of dynamical events: particle motions and extended body motions. These two classes of events are distinguished by the number of object properties of dynamical relevance to the motion context. For particle systems, only the motion of the object's center of mass is relevant to its dynamics, whereas for extended body systems, mass distribution, orientation, rotation, and other properties are dynamically relevant variables. It is important to keep in mind that the relevance of object properties depends not on the object itself but on the motion context in which the object is observed.

Dynamical analyses of particle motions are much simpler than are those of extended body motions. This is due to the increased number of variables that must be included in an adequate dynamical representation of extended body events. Particle motions can always be represented by equations that relate only one category of information: position over time. In essence, particle systems can be understood in terms of center-of-mass displacements. Dynamical representations of extended body motions always relate more than one category of information. In extended body motions, it is not sufficient to know where an object's center of mass is located; rather, such relational properties as mass distribution—how much of the object's mass is located where—must be appreciated. The relating of different categories of information is performed through multiplicative processes and results in multidimensional quantities that are not categories of perceptions.

The definition of dimensionality that we have provided was obtained from physics, and thus does not serve to define dimensionality in perception. With regard to human performance, there are two questions. The first is, What configural and kinematic patterns can be distinguished to form clear and identifiable dimensions? The second is, Which of these perceptual dimensions will be construed to be relevant for dynamical judgments? These questions are far from being answered. Be this as it may, our account, drawn from a physical analysis of dimensionality, defines limits on human performance. It has been shown for a variety of situations that people tend to treat multidimensional problems as unidimensional ones (Shepard, 1964). Extending this finding to dynamical contexts implies that dynamical intuitions must suffer as the boundary is crossed between particle and extended body motion contexts.

People's commonsense understandings are fairly good for particle motions. Although people sometimes make erroneous predictions due to their misrepresenting the dimensionality of these simple systems, their dynamical judgments are quite accurate when they are actually observing the ongoing events. Animation often segregates event dimensionality in time.

When people attempt to form dynamical understandings of extended body motions, dynamical competence begins to break down. when asked to reason about simple balances, people perform about as well as when asked difficult particle motion problems; however, when people are asked to predict the behavior of rolling wheels, almost no one anticipates that mass distribution will affect the rate at which a wheel rolls down an inclined plane. Observing the complex object motions found in tops and gyroscopes produces in people a perceptual catastrophe that is experienced as wonder. Perception in such situations informs people that forces exist that they cannot appreciate. Tops and gyroscopes are wonderful, in part, because in perceiving their apparent gravity-defying behavior, people become aware of their own perceptual limitations.

Gravity is, of course, also wonderful in this sense; it acts as an invisible force. However, the perceived effect of gravity is profoundly different in particle motions and extended body motions. Dropping a spinning top and a nonspinning top off a tower will produce equivalent falling trajectories that are easily assimilated by common sense. Placing these objects on a pedestal results in quite different motions: The nonspinning top falls down, whereas the spinning top falls sideways, and thereby precesses. The latter event cannot be assimilated by common sense.

There are people, physicists, who have a dual awareness of the characteristics of mechanical systems. This awareness schism is quite interesting to observe and is easily elicited. Most of the problems discussed above were presented to a group of 19 high-school physics teachers, individuals who, more than any other group, are responsible for explaining the elementary principles of classical mechanics to naive students. These teachers were forced to answer the questions fairly rapidly, and were thereby prohibited from generating the formal representation for each problem that would allow for an analytical derivation of the correct answers. The performance of this group was no better than that found for the tested undergraduates or university physics professors interviewed under similar time constraints.

Prevent competent physicists from making explicit calculations about such events as rolling wheels, and they exhibit the basic confusions that are found in naive observers; they are generally aware that mass is not relevant, as the equivalence principle (that all objects are accelerated at the same rate regardless of mass) is second nature; however, they are often not so sure about radius and mass distribution. However, if given time, most physicists could work the problem out in a few minutes. The average physicist will then inform you that the rolling wheel problem is trivial. This is the second awareness: the formal understanding of the mechanical system.

It is the goal of physics educators to encourage in their students the development of this second awareness. The intuitive physics literature has been influential in framing theories of what constitutes the learning of physics. In particular, it has been frequently suggested that physics instruction should take into account the naive beliefs that students bring to the learning situation (Carey, 1986; Champagne et al., 1980, Clement, 1982, McCloskey, 1983a; Reif 1986). We believe that this prescription is likely to be misapplied. Our preliminary investigations suggest that for complex, extended body motions, people's dynamical understanding failures are not due to their holding erroneous theories; rather, these failures result from intrinsic limitations in processing dynamical information. When presented with the rolling wheel problem, people are more often muddled than misguided. Moreover, physicist and physics teachers share with naive individuals a sense of befuddlement regarding the extended body motions that we have examined. We propose that the adequacy of commonsense dynamical judgments depends on the degree of dimensionality that is both inherent in the physics of the event and presumed to be present by the observer.

What, then, constitutes learning physics? And what is going on when a physicist spends a quarter of an hour working out a problem and then tells you that it is trivial? We do not propose a theory of learning in this article, but we do offer the following idea: Learning physics is the transportation of commonsense notions of symmetry and simplicity to the mathematics that describe dynamical events. In this sense, learning physics is concerned with a change in the domains of understanding: a shift from the phenomenal world to the formal world captured by the calculus of variations. What makes the rolling wheel problem trivial is that its mathematical structure is very simple; the manifest symmetries between the rotational and translational degrees of freedom are easily displayed in a simple and compact notation. Furthermore, the equation of motion can be solved analytically in terms of elementary functions and integrals. Even if it is not possible to see what is going on with a rolling wheel, it is easy to see what is going on with its mathematics. What are common to commonsense and the formalisms of physics are their inherent symmetries, symmetries that form the basis for their intelligibility.

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Appendix

A Minimum Principle for Natural Motions

All mechanical systems are put on equal footing by abstractly representing them as a single point in a multidimensional configuration space. The dimension of the space is the number of coordinate specifications that have to be made to place and orient every object in the system exactly. In general, there are six degrees of freedom that a rigid body has in its placement and orientation. Three spatial coordinates are required to place the center of mass of an object unambiguously, and three angles are required to specify the orientation of the principle axes. A mechanical system containing N objects will therefore require at most $6N$ coordinate specifications to specify its instantaneous state; that is, the configuration space has $6N$ dimensions. As the dynamical system evolves in time and the objects in the system take on new coordinate values, the system point traces out a path in the configuration space.

The configuration space can also be thought of simply as an abstract multidimensional space. A point in this space can be thought of simply as a vector, that is, as an ordered list; alternatively, a point can be thought of as the state of a mechanical system. In the same sense, a path in the configuration space may represent the evolution of a mechanical system, or the path may be simply the graph of a continuous function. Not all paths in the configuration space are generated by mechanical systems. In fact, most paths represent impossible motions, that is, motions that are inconsistent with Newton's laws. Between any two points in the configuration space there is at most one path that could be generated by a mechanical system. Such a path has a special property; it minimizes a quantity known as the *action*. (For a rigorous development of these ideas, see Goldstein, 1981, or Landau & Lifshitz, 1982. For a more leisurely treatment of minimum principles in nature, see Hildebrandt & Tromba, 1986.) The action is a path integral and is therefore a global property of each curve in the configuration space. For each path a different action is associated. The path on which a mechanical system moves is that path with the least action. The evolution of a mechanical system is a single example of a much more general class of problems that are treated by the calculus of variations (see Courant, 1962). In this branch of mathematics one attempts to identify curves and surfaces that possess some sort of global minimum property. The property being considered can be quite general. As an illustrative example, consider the problem of a hanging chain suspended between two points. What shape will it naturally assume? There are an infinite number of possible chain shapes, but there is only one shape that minimizes the total gravitational energy. When the problem is worked out to minimize the total energy, it is found that the chain shape describes a curve called a *catenary*. The dynamics of mechanical systems are treated in essentially the same way, but instead of energy being minimized, a related quantity, action, is minimized. (Action has the dimensions of Energy × Time.) From a mathematical point of view, the problem of describing a chain shape differs from the description of a dynamical system only in the nature of the quantity that is minimized. The problems are posed in the same way.

Received April 22, 1988
Revision received August 22, 1988
Accepted August 22, 1988 ■