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Understanding Collision Dynamics

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In two experiments we investigated people's ability to judge the relative mass of two objects involved in a collision. It was found that judgments of relative mass were made on the basis of two heuristics. Roughly stated, these heuristics were (a) an object that ricochets backward upon impact is less massive than the object that it hit, and (b) faster moving objects are less massive. A heuristic model of judgment is proposed that postulates that different sources of information in an event may have different levels of salience for observers and that heuristic access is controlled by the rank ordering of salience. It was found that observers ranked dissimilarity in mass on the basis of the relative salience of angle and velocity information and not proportionally to the distal mass ratio. This heuristic model was contrasted with the notion that people can veridically extract dynamic properties of motion events when the kinematic data are sufficient for their specification.

The abilities that people have in reasoning about natural motions are in part determined by the number of independent categories of information that physically describe the event. A theory of dynamic event complexity has been given by Proffitt and Gilden (1989) in which it is argued that people are capable of accurate dynamic judgments only when the event is so simple that a correct judgment can be based on a single category of information. This theory proposes that there are fundamental perceptual limitations that prevent organization of different categories of information into synthetic multidimensional quantities.

Categories of information, or dimensions, in Proffitt and Gilden's theory are the collection of object properties (e.g., spatial position, mass, shape, size, and orientation) that determine the way in which the object moves. The dimensionality of an event is determined by the motion context in which objects appear as well as by the objects themselves. An example illustrates this notion of dimensionality. Figure 1 depicts a wheel in two motion contexts that differ in their dimensionality. The first context is the free-fall motion of a wheel. In this context there are no object properties of the wheel that are relevant to its motion in a gravitational field. All wheels, regardless of their size, shape, and mass will fall in exactly the same way. The free-fall speed of any wheel depends only on the position of its center of mass, relative to the height from which it is dropped. In this context, the difference between the instantaneous position of the center of mass and the initial dropping height appears as the single category of information that specifies the wheel's motion. This event is one dimensional. In the second context, the wheel rolls down an inclined plane. There are now two categories of information that determine the wheel's speed: the position of the center of mass and the moment of inertia that the wheel has about its center of mass. This moment is determined by the ratio of the inner and outer diameters. Wheels that are rimlike roll down the ramp slower than wheels that are solid throughout. Mass and absolute size have no relevance to the motion in either context.

The abilities that people have in reasoning about wheels depend critically on the motion context in which the wheel is presented. Most people know how things drop, although terrestrial experience (more massive objects generally have greater terminal speeds in resistive atmospheres) and the failure to distinguish between force and acceleration do induce a bias toward supposing that heavy objects drop faster. People's understanding of the rolling context is much different (Proffitt, Kaiser, & Whelan, 1988). People often suppose that the irrelevant dimensions of absolute size and mass have dynamic influences. Moreover, there is a high agreement that mass distribution (moment of inertia), the only physically relevant parameter, is irrelevant. Inability to reason about rolling wheels extends even to bicycle racers, high school physics teachers, and college professors of physics (Proffitt, McAfee, & Hecht, 1989). Although members of the latter two groups can, given sufficient time, solve rolling-wheel problems on the basis of the equations of motion, their formal understanding does not translate into immediate and accurate impressions of the sort that are encountered for wheels that are dropped.

We have tested this theory of dynamic event complexity in a number of physical environments. Assessed performance in reasoning about such objects as balances and tops (Proffitt & Gilden 1989; Proffitt et al. 1989) as well as about Archimedes' principle (Whelan, 1987) has consistently shown that, in general, people reason adequately about objects only in one-dimensional motion contexts. Performance in multidimensional reasoning tasks ranges from mere inaccuracy (balances) to amazement (tops). Although each environment has idio-

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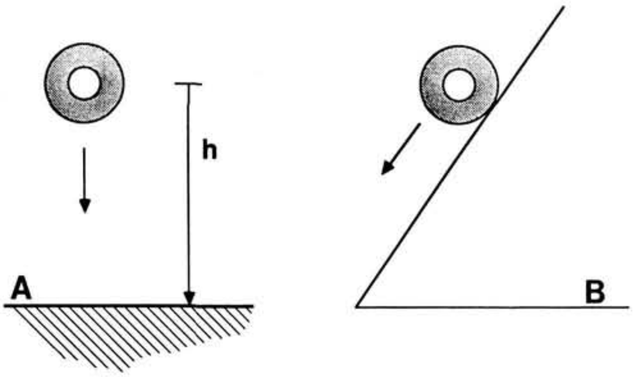

Figure 1. Two motion contexts for the wheel. In Panel A the wheel is shown in free fall. There is no attribute of the wheel that affects its velocity. A free-fall context treats all objects as being extensionless point particles located at their center of mass. In Panel B the wheel is shown perched on an inclined plane. The mass distribution, parameterized by the ratio of inner and outer diameters, influences the rate at which the wheel rolls. (h = height.)

syncracies that make the reasoning tasks more or less difficult, the transition in complexity from one to many dimensions of information appears to be crucial for human understanding.

There are a couple of apparent counterexamples to the claim that people are unable to adequately judge events that require the combination of two or more dimensions of information. Foremost among these is the sensitivity that people have to the relative mass of two balls in a collision event. The mass ratio is a dynamic invariant that is specified by two independent categories of information in the proximal pattern of motion. These categories are given by the decomposition of the velocity vectors into their component parts. The velocity vectors are specified by their direction (angle information) and their magnitude (speed information).

The synthesis of angle and speed information that is required for the judgment of mass ratio is formally described by the conversation laws for energy and momentum transfer (see the Appendix for a full discussion of the physics of collisions). The multidimensional quantities that arise from the conservation equations are the *projections* of the velocity vectors onto, respectively, the collision axis and an axis orthogonal to it. These projections are the rate of drift that the balls have, not along their trajectory, but with respect to coordinate axes. Projections are intrinsically multidimensional quantities in the sense that we have been developing. In order to form a projection of a vector, both angle and magnitude information has to be combined. Formally, this combination is accomplished by the dot product. The projection of a vector **A** onto vector **B** is computed as

$$(\mathbf{A} \cdot \mathbf{B})/|\mathbf{B}| = |\mathbf{A}| \cos \alpha,$$

where α is the angle between **A** and **B**. In the context of collisions, we are interested in the magnitude of the velocity projections on the axis specified by the trajectories of the incoming balls and the axis orthogonal to it. This equation demonstrates exactly how angle and speed information have to be combined in order for mass ratio to be abstracted from the proximal motion pattern. What an observer must accomplish in viewing a collision in order to make veridical mass ratio judgments is to *distinguish speed along a trajectory from speed relative to a dynamically relevant axis.*

Runeson (1977) noted that there is a one-to-one correspondence between the physical variables in the conservation laws and the available information in the kinematic motion patterns. On this basis he predicted that observers of collisions would not only be able to distinguish which of two balls was heavier, but would also be able to correctly rank order the mass ratios from an ensemble of collision events. Kaiser and Proffitt (1984) performed a set of experiments that assessed this ability and concluded that people do in fact possess a sensitivity that allows the dimensional combination required for mass ratio estimation. Further studies indicated that human observers are able to distinguish natural from unnatural collisions (Kaiser & Proffitt, 1987), a competence that is consistent with the thesis that people can effectively construe the multidimensionality that is intrinsic to collisions.

Runeson and Frykholm (1983) extended the notion that dynamic invariants can be abstracted from kinematic motion patterns in a theory of kinematic specification of dynamics (KSD). The KSD theory essentially consists of two statements. First, when the kinematic data are sufficient, people will be able to make veridical judgments in circumstances that have a high degree of dimensional complexity. Second, the dynamic impressions that are attendant to the perception of motion events are not mediated by inference or by solving equations of motion for specific dynamic invariants. Instead, the perception of dynamical invariants is direct; dynamic invariants are available in perception because people have special sensitivities to them. In KSD theory the goal is therefore to describe how perceptual systems are structured to manifest the required sensitivities.

The second line of evidence supporting KSD theory is the apparent ability that people have in estimating the weight of objects lifted by another person (Bingham, 1987; Runeson & Frykholm, 1981). This regime of observer sensitivity is especially difficult to evaluate because the nature of the kinematic data that provide perceptual support has not been clarified. In the absence of a kinematic theory, it is not possible to determine the dimensionality of the quantities that are required to abstract the weight invariant. Furthermore, it is not clear whether this is a general ability or one that is specific to the experimental designs that have been developed. We return to these issues in the General Discussion.

Collisions, in contrast to weight lifting, have been exactly described at both a kinematic (motion patterns) and dynamic (forces and accelerations) level. The behavior of two balls in a rigid body collision is uniquely determined by the geometry of the collision and the conservation equations for energy and momentum. Collisions, thus, provide a unique laboratory for investigating how people assimilate multiple categories of information. Most important, one can specify beforehand the detailed structure of the parameter space that contains all possible collisions. It is this critical feature that permits experimental designs whereby a superficial ability that exists only for a limited regime of stimuli can be distinguished from a general competence.

In this article we reexamine people's abilities to judge mass ratios in collision events by sampling from a large region of the collision parameter space. Two theories of judgment are contrasted. The first is the KSD theory. This theory has found empirical support from incomplete studies (Kaiser & Proffitt, 1984, 1987) that have sampled the collision parameter space only in limited regions. The second theory is that mass ratio judgments are made on the basis of heuristics that lead to veridical judgments only in isolated circumstances. Preliminary evidence for a heuristic theory has been developed by Todd and Warren (1982), who analyzed the degenerate class of one-dimensional collisions.

Two Models of Motion Analysis

There are two competing theories of people's abilities to make relative mass judgments from the kinematic information available in a collision event. These theories are roughly as follows:

1. The KSD theory. People are sensitive to the appropriate physics and therefore should be able to make dynamic judgments independent of specific collision parameters.

2. The heuristic theory. People are not sensitive to dynamic invariants but do have some commonsense notions about how the world operates and make dynamic judgments on the basis of heuristics that articulate these notions. As in any heuristic analysis, these judgments are expected to be correct only in a limited regime of collision parameters.

The first experiments that attempted to distinguish between these two models were conducted by Todd and Warren (1982). They showed that for head-on collisions, observers can reliably distinguish which object is heavier only when a heavy object hits one that is lighter. When a light object hits a heavier one, observer performance is variable, improving monotonically with the disparity in mass. Todd and Warren concluded that KSD principles cannot be correct in this regime because people clearly do not have a general ability to judge mass ratio. They further argued that observers appeared to be using heuristics, although it was not clear what these heuristics were. One heuristic that was isolated was that faster moving balls appear to be lighter. Todd and Warren's data are discussed in detail in discussion of the second experiment.

A proper analysis of the structure of mass ratio judgments cannot be accomplished unless observers are allowed to view collisions in their most general form. Collisions in two dimensions are fundamentally different from the special class of one-dimensional collisions, which are necessarily head-on. In a head-on collision, at no single moment in time is there sufficient information available to make a judgment of mass ratio. One must combine the postcollision velocities, which one can see, with what one remembers of the precollision velocities (see the Appendix for a detailed discussion of the relevant equations). Two-dimensional collisions, however, provide an opportunity for assessing mass ratio from quantities that are simultaneously available in the event. This opportunity arises because momentum must be conserved not only along the collision axis (the axis along which the balls approach), but also along an axis that is orthogonal. The rates at which the balls drift away from the collision axis (the velocity projections on the axis orthogonal to the collision axis in the postcollision epoch) provide sufficient information for the specification of mass ratio (see Equation A-6 in the Appendix). Formally, the judgment of mass ratio in a two-dimensional collision does not require any information from the precollision epoch.

Extensive pilot studies we conducted on observer competence in mass ratio judgment indicated that people do not spontaneously form the required velocity projections. Instead, observers treated the angles of the trajectories as one category of information, and the speeds along the trajectories as a second. We found that observers seemed to be using two heuristics, one for each category of information.

1. The angle heuristic. The ball that scatters at the greatest angle appears to be lighter.

2. The velocity heuristic. The ball with the greatest velocity in the postcollision epoch appears to be lighter.

These two heuristics are consistent with commonsense notions that arise in everyday experience.

A second finding from our pilot studies was that the two different categories of information were evaluated independently. For example, collisions in which the incoming ball ricocheted were so striking that observers tended to ignore the information available regarding the relative speeds. Alternatively, when both balls scattered forward, observers seemed to be basing their judgments on the speed information alone. These observations implied that the *salience* of information had to be taken into account in describing the logic of heuristically based judgment.

Overview of the Studies

We have discussed two accounts for the processing that occurs when people base dynamic judgments on kinematic data. The following experiments were designed to evaluate which of these accounts is supported by data drawn from a large ensemble of collisions in two dimensions.

Both KSD and the heuristic theory can be based solely on information available in the postcollision epoch. Experiment 1 evaluated the sufficiency of information in this epoch for the judgments that people do in fact make. This experiment also allowed us to determine the circumstances in which observers' impressions of relative mass are accurate. Experiment 2 probed a large region of the collision parameter space in order to clarify the manner in which collision information is treated in the judgment process. We were especially interested in analyzing the role of salience in heuristic analysis. In this experiment we also examined more closely what people in fact assess when asked to judge the mass ratio per se.

Experiment 1: The Information Used in the Perception of Relative Mass

In a general two-dimensional collision the mass ratio is determined by a threefold redundancy in the kinematic information present in the motion pattern. The relative mass specified by a collision event can be determined from any of three conservation equations (see Appendix). Runeson (1977) and Todd and Warren (1982) focused on the conservation of x-momentum (Equations A-1 and A-5), although the conservation of energy (Equations A-3 and A-7) can equally well serve as the basis of a smart perceptual device that computes mass ratios. In the generalization to two dimensions, y-momentum conservation (Equations A-2 and A-6) also is available for the specification of the mass ratio. The y momentum conservation equation is unique in that it does not refer to the velocity of the incoming ball, and it allows the computation of mass ratio from quantities that are simultaneously available in the event. The precollision velocities are formally superfluous for the computation of the mass ratio. In Experiment 1 we determined whether the velocity of the incoming ball was perceptually required for the judgment of mass ratio. In this way we could determine the necessity of a memory reconstruction of the precollision epoch and pinpoint which of the conservation equations are sufficient.

Method

Subjects. Twenty-four observers (12 male and 12 female) participated. They were recruited from introductory psychology courses and received course credit for their participation. None of the observers was aware of the study's purpose.

Stimuli. The experiments concerned the collision of one rigid ball with another that has the same radius and is initially at rest in the observer's frame of reference. External forces such as friction are presumed to be absent. Figure 2 shows the geometry for this type of collision. Ball 1 is taken to be impinging, and Ball 2 at rest. The

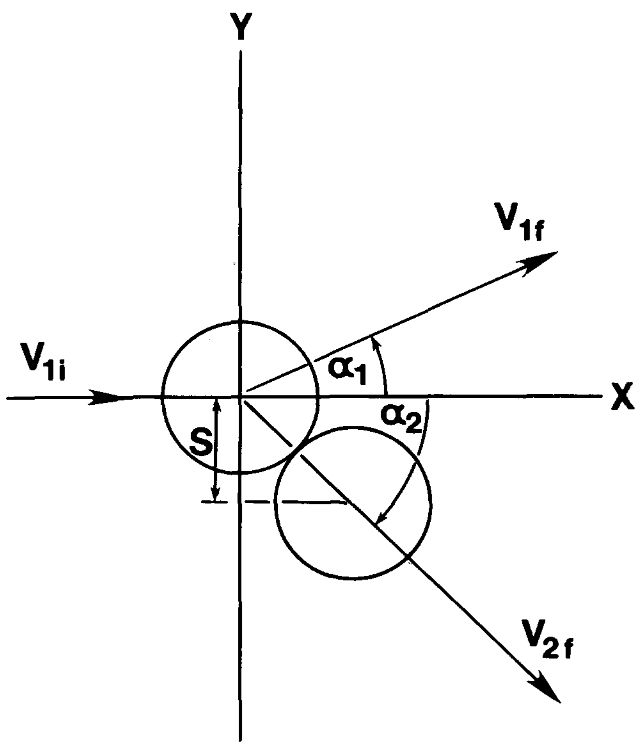

Figure 2. The geometry of a general collision between two balls of equal diameter. The collision is shown in a frame of reference in which one ball impacts on another that is initially stationary. (v_{1i} = incoming velocity of Ball 1, s = impact parameter, α_1 = angle of exit of Ball 1, α_2 = angle of exit of Ball 2, v_{1f} = outgoing velocity of Ball 1, v_{2f} = outgoing velocity of Ball 2.)

collision dynamics are most easily written by defining a coordinate system in which the x-axis is identified with the trajectory of the incoming ball. The y-axis is orthogonal. The most general collision between objects of any shape and size is intrinsically two-dimensional. The parameters of the collision are as follows:

m_1: mass of Ball 1,
m_2: mass of Ball 2,
Q: mass ratio m_1/m_2,
v_{1i}: incoming velocity of Ball 1,
v_{1f}: outgoing velocity of Ball 1,
v_{2f}: outgoing velocity of Ball 2,
α_1: angle of exit of Ball 1 (opens counterclockwise),
α_2: angle of exit of Ball 2 (opens clockwise),
s: impact parameter,
D: diameter of each ball.

All stimuli were viewed on a Tektronix 4129 3-D Color Graphics Workstation with a high resolution (4 lines/millimeter) display screen, 30 cm high by 35 cm wide. The collision event took place in a circular window centered on the screen center with radius of 12.5 cm. Inside the window, the background was illuminated with a uniform light gray. Outside the window, the screen was black. The purpose of the window was to remove any horizontal–vertical interactions that might be induced by the rectangular geometry of the physical display screen and thus to allow the faster ball to leave the screen first. The colliding balls were depicted on the display screen as untextured disks, resembling more closely hockey pucks than three-dimensional balls. The diameter of the balls was 1.2 cm in all cases, and in each event the balls were assigned two different colors randomly.

The collision events were depicted in two different ways. In the nonoccluded condition the event would be initiated with one ball appearing at the screen center at rest, and the other appearing at a leftward displacement of 8.3 cm, moving at either 6.7 or 10.1 cm/s. Two short beeps signaled the onset of a collision. The first beep occurred 500 ms before impact, and the second beep occurred at the moment of impact. The occluded condition was similar except that the impinging ball was invisible until the moment of contact and the sounding of the second beep. In the occluded condition, the impinging ball was replaced by a rectangular shaded area 3.5 × 15.8 cm, centered along the path the impinging ball would have taken. The rightmost boundary of the occluding area was tangent to the stationary ball. The presence of the occluding area made the event appear as if the unseen ball was continuously present, although unseen in the precollision epoch, as opposed to appearing to come into existence at the moment of contact. The occluding area was removed at the onset of the postcollision epoch. In both types of collisions the balls would disappear as they left the illuminated circular window. The collision display was terminated only after both balls had exited the circular infield.

The collision parameters are given in Table 1. The parameter space of possible collisions in two dimensions is quite large. To create a tractable initial experiment we chose two slices through this space: α_2 = 20°, and α_2 = 30°. These choices were equivalent to confining the collisions to the impact parameters s/D = .34 and .50, respectively.

All collisions were viewed in four different variations. Two initial speeds of the incoming ball were used, in order to prevent observers from anticipating the moment of collision in the occluded condition. In addition, the mirror-image collisions (taken by reflection through a plane containing the collision axis and orthogonal to the screen) were viewed. Pairing each collision with its mirror image allowed us to average over any perceptual distortions that might have been created by a perceived or imagined up/down anisotropy. An example of such an anisotropy would be an imagined gravitational field (Kaiser & Proffitt, 1987).

Design. A mixed-model factorial design was employed, with α_2 (or impact parameter) as a between-subjects factor, and mass ratio, occlusion/nonocclusion, speed of impinging ball, and up/down ge-

Table 1
Collision Parameters for Experiment 1

m_1:m_2	$\alpha_2 = 20°$			$\alpha_2 = 30°$		
	v_{1f}	v_{2f}	α_1	v_{1f}	v_{2f}	α_1
2:1	4.6	12.5	28	5.8	11.5	30
3:2	3.9	11.3	41	5.3	10.4	41
4:3	3.7	10.7	48	5.2	9.9	46
1:2	4.6	6.3	110	5.8	5.8	90
2:3	3.9	7.5	99	5.3	6.9	78
3:4	3.7	8.1	93	5.2	7.4	72

Note. All velocities are given in units where $v_{1i} = 10$. m_1 = mass of Ball 1, m_2 = mass of Ball 2, v_{1f} = outgoing velocity of Ball 1, v_{2f} = outgoing velocity of Ball 2, α_1 = angle of exit of Ball 1, α_2 = angle of exit of Ball 2.

ometry as within-subject factors. As mentioned above, there were six levels of mass ratio and two levels for all other factors.

Procedure. The experiment began with a brief description of the collision stimuli. The observers were asked to indicate which of the two balls was heavier by moving a cursor along a scale. The scale was divided equally into two colors corresponding to the colors of the balls that exited above or below the collision axis. The orientation of the scale was also consistent with the exit trajectories; that is, if an observer thought that the ball that exited toward the top of the screen was heavier, he or she moved the cursor upward. Observers executed marker placement by rotating a thumbscrew on the Tektronix keyboard. If the balls appeared to be of equal mass, marker placement would be at the midpoint of the scale. Twelve practice trials were viewed that were representative of the collisions in the experimental trials, but were not identical to them. The experimental session consisted of judgments on 48 collisions.

Results and Discussion

The data in this experiment were analyzed in terms of whether the observers were correct in their judgment of which ball was heavier. An overall analysis of variance showed that there was no significant difference between α_2 groups (recall that the α_2 groups differed in the exit angle of the struck ball), $F(1, 22) < 0.01$, $p < 0.99$. Similarly, the velocity of the incoming ball was not a significant factor $F(1, 22) = 1.21$, $p < 0.28$; nor was there a significant difference between collisions and their mirror images, $F(1, 23) = 0.33$, $p < 0.57$. Both mass ratio, $F(5, 110) = 34.76$, $p < 0.0001$, and occlusion, $F(1, 22) = 5.49$, $p < 0.029$, were significant factors, as was their interaction, $F(5, 110) = 8.06$, $p < 0.0001$. No other main effects or interactions were significant.

Figure 3 shows the percentage correct for each mass ratio in the two occlusion conditions averaged over velocity, mirror image presentation, and α_2 group. As Figure 3 illustrates, performance in the assessment of which ball was heavier was strongly influenced by whether the heavier ball was impinging ($m_1 > m_2$). When $m_1 > m_2$, observers were generally quite accurate in their assessment of which ball was heavier, although observers were significantly better *if they could not see the incoming ball*, $F(1, 23) = 15.84$, $p < 0.0001$. Although perhaps owing to a ceiling effect, observer accuracy, when $m_1 > m_2$, was independent of mass ratio, and in particular the extreme mass ratio of 2:1 was not judged with better accuracy, $F(1, 23) = 0.60$, $p < 0.44$. Alternatively, when a light ball impinged on a stationary heavier one ($m_1 < m_2$), observers' accuracy was significantly better when they could see the

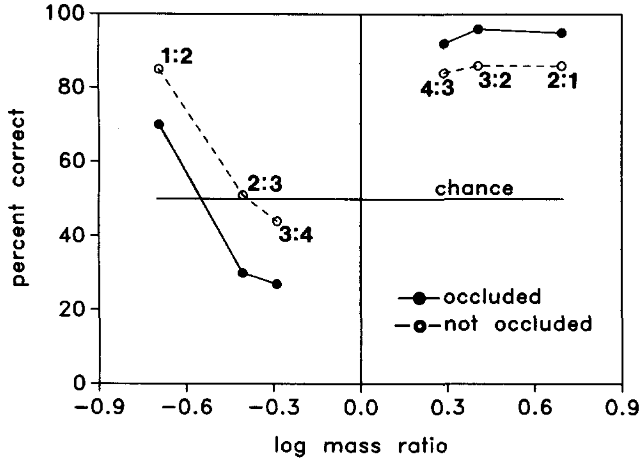

Figure 3. Results for Experiment 1. The percentages correct of observers' judgments of which of two balls was more massive are plotted versus the mass ratio. Two conditions are shown: An occluded condition in which the incoming ball was not visible until the moment of impact, and a nonoccluded condition in which the entire collision event was displayed.

incoming ball, $F(1, 23) = 49.23$, $p < 0.0001$. Furthermore, when $m_1 < m_2$, observer performance was quite sensitive to mass ratio, and accuracy was significantly better for the extreme mass ratio of 1:2, $F(1, 23) = 211.4$, $p < 0.0001$.

The differences in performance that exist between the two occlusion conditions must be understood in the context of the small differential magnitudes that were in fact found. First, the functional appearance of the curves in Figure 3 is very similar. The interaction that exists between mass ratio and occlusion was generated by the crossing in performance at the transition from $m_1 < m_2$ to $m_1 > m_2$. Within the two mass ratio groupings no significant interactions of mass ratio with occlusion were detected. Second, the performance magnitudes are not that different. The mean performance averaged within the $m_1 < m_2$ group, within the $m_1 > m_2$ group, and over all collisions is given in Table 2. There was an overall 5% improvement in performance when the impinging ball was visible. This increment was due entirely to observers performing no worse than chance when $m_1 < m_2$.

Several conclusions can be drawn from Experiment 1. The first is that observers do not veridically assess mass ratio in general. As Todd and Warren (1982) also found, observers were quite accurate when $m_1 > m_2$, independent of the exact value of the mass ratio, and they were variably accurate when $m_1 < m_2$; approaching 80% accuracy for $m_1/m_2 < 1/2$. Evidently, observers were not directly assessing mass ratio in

Table 2
Overall Occlusion Effects

Condition	$m_1 < m_2$	$m_1 > m_2$	All
Occluded	0.42	0.94	0.68
Nonoccluded	0.60	0.87	0.73

Note. Results are given as percentage correct. m_1 = mass of Ball 1, m_2 = mass of Ball 2.

the sense of KSD, for the kinematic information is sufficient in all cases.

The second conclusion that we draw is that the precollision epoch is not required for the determination of mass ratio. In the regime in which observers were good, $m_1 > m_2$, they were better if they could not see the precollision regime at all. Where observers were inaccurate ($1 > m_1/m_2 > 1/2$), viewing the precollision epoch raised performance to at most a chance level. Observers apparently did not use their memory of the speed of the incoming ball in arriving at mass ratio judgments.

The results of Experiment 1 are consistent with heuristic usage. The two regimes of mass ratio were distinguished by the correctness of the heuristic that the faster ball is lighter. For the stimuli in this experiment, when $m_1 > m_2$ this heuristic was in fact true, but when $m_1 < m_2$, this heuristic led to a false conclusion. The data for the occluded condition suggest that this heuristic was used exclusively, leading to almost perfect performance when $m_1 > m_2$, but also leading to worse than chance performance when $1 > m_1/m_2 > 1/2$. The observers in the nonoccluded condition showed a more variable performance, indicating that they were using a second heuristic. The enabling of this second heuristic did not appear to be related to memory for the speed of the incoming ball, but rather to the opportunity for witnessing ricochet in a continuous, ongoing event.

The occluded and nonoccluded conditions were primarily distinguished by performance in the $m_1 < m_2$ regime. Here the ball that scattered with the greatest angle was the lighter, whereas the faster moving ball was heavier. The significant differences that were found between the two conditions are interpretable in terms of the nonoccluded group using an angle heuristic. Observers were best when there was a large angle of ricochet. Now observers in the occluded condition could not see the ricochet as an ongoing perceptual event, and it may be for this reason that they did not find the ricochet sufficiently salient to allow it to influence their judgments. In this single sense, the precollision epoch informs the judgment of relative mass. The precollision data did not enter quantitatively in terms of incoming speed, but rather as a basis for the perception of ricochet.

There were several stimuli in Experiment 1 that had ricochet and for which subject performance was at a chance level. An informal analysis of ricochet collisions indicated that when the velocity ratio v_{2f}/v_{1f} was large (in excess of 1.5) roughly half the observers consistently rated the heavier, but faster moving, ball as lighter. This behavior suggests that when both the angle and the velocity ratios are sufficiently salient observers split in their heuristic usage. The following model was developed in order to formalize the way in which observers seemed to be using salience in accessing heuristics. This model then provided the theoretical structure for the second experiment, in which the logic of heuristic analysis was clarified.

A Formal Heuristic Model

A theory of judgment that incorporates heuristics is not complete without rules that govern the circumstances in which the heuristics are used, for there will be cases where the heuristics lead to conflicting judgments. An additional notion is required for the decision of when to use a given heuristic. We suggest that observers base their judgments on that category of information that is most salient to them. Formally, we define a salience function on two categories of information in the following way. Each category of information, p and q, is regarded as having some level of salience for the observer. This salience level is denoted by a function $S(\text{p})$. The function S does not necessarily induce a metric, although it might, and in this article we do not regard distances in the space of salience to be placed on an interval scale. Rather, S induces an ordering only in the sense that $S(\text{p})$ may be greater than or less than $S(\text{q})$. A function that can be constructed on pairs of points in the space of saliences is the theta function. It is defined as

$$\theta[S(\text{p}) - S(\text{q})] = \begin{cases} 1 & \text{if} \quad S(\text{p}) \geq S(\text{q}) \\ 0 & \text{if} \quad S(\text{p}) < S(\text{q}) \end{cases}. \tag{1}$$

The heuristics in this scheme are defined on only one category of information. This is not required theoretically, but displays the way information is in fact used by people; that is, the heuristics that people use mention only one category of information. This is consistent with the theory of dynamic event complexity described by Proffitt and Gilden (1989) and captures the way observers seemed to be treating the information in collisions in Experiment 1. Heuristics enable observers to evaluate events that present multiple categories of information through isolation on single dimensions. Salience here operates as the psychological medium in which different categories of information (angles vs. velocities, say) are compared.

In this model we analyze only the binary judgment of which ball is heavier. The two categories of information, p and q, refer to comparisons made on the angles and speeds of the exit trajectories. The numerical value assigned to heuristic $H(\text{p})$ is unity if using the heuristic defined on p leads to a correct judgment, and zero otherwise. Our decision model is then written,

$$J(\text{p, q}) = H(\text{p})\theta[S(\text{p}) - S(\text{q})] + H(\text{q})\theta[S(\text{q}) - S(\text{p})], \tag{2}$$

where $J(\text{p, q})$ is a binary judgment on two variables. $J(\text{p, q})$ is unity, is a correct decision, if the category of information most salient to the observer leads to using a heuristic that is in fact true under the circumstances that obtain in reality.

The heuristic theory that we propose does not predict that observers will always make veridical judgments. Rather, observer accuracy will be limited to those regions in the collision parameter space where the salient features of a collision coincide with heuristics that happen to be true. For this reason we regard the use of heuristics as requiring a statistical treatment of the decision process. The ensemble averages (denoted by < > and taken over observers) of equation 2 that describe normative observer accuracy are developed as follows:

$$\langle J(\text{p, q})\rangle = H(\text{p})\langle\theta[S(\text{p}) - S(\text{q})]\rangle + H(\text{q})\langle\theta[S(\text{q}) - S(\text{p})]\rangle, \tag{3}$$

where averages are not taken over the heuristics, because it is assumed that all observers use the same heuristics and differ only in the circumstances in which they use them.

Mean subject accuracy is reduced in this model to an analysis of mean salience. There are four distinct regimes of accuracy that are of particular interest.

1. $H(p)$ is in fact correct, $H(q)$ is in fact false, and $<\theta[S(p) - S(q)]> = 1$. In this case $<J(p,q)>$ approaches unity—veridical judgment.
2. $H(p)$ is in fact false, $H(q)$ is in fact correct, and $<\theta[S(p)-S(q)]> = 1$. In this case $<J(p, q)>$ will approach zero—erroneous judgment.
3. $H(p)$ is in fact true, $H(q)$ is in fact false, and $<\theta[S(p) - S(q)]> = <\theta[S(q) - S(p)]> = 1/2$. In this case $<\theta[S(p) - S(q)]>$ approaches 1/2, guessing at chance.
4. $H(p)$ and $H(q)$ are both true. In this case $<J(p, q)>$ should approach unity independent of salience.

This formal model provides a number of testable hypotheses. With the exception of Equation 2, all of the listed circumstances can arise in natural collisions. In Experiment 2 we analyzed in some detail the logic of salience and heuristic access.

Experiment 2: Heuristic Analysis as a Basis for Judgment

The central concept in our model is that the category of information that is most salient will lead to the use of a heuristic based on that information. In the postcollision environment there are two available categories of information: (a) the relative exit speeds and (b) the relative size of the angular departures from the vector specified by the incoming ball. Access to the heuristics based on this information (faster implies lighter, as does larger scattering angle) will depend on how the proximal motion patterns are psychologically mapped into salience.

We do not have a fundamental theory of angle and velocity salience that specifies the mapping between motion and salience. Experiment 1 suggested that the presence of ricochet ($\alpha_1 > 90°$) provides salient angle information and that speed ratios in excess of 2 provide salient velocity information. For the purpose of demonstrating the key features of a heuristic based theory, these values were used to set the salience boundaries.

Figure 4 illustrates the basic collision structures that test our theory of salience-driven heuristic access. In this figure the lengths of vectors represent velocity magnitude and the two paths are differentiated by the line widths. There are essentially two levels of salience for each category of information and this generates a 2 × 2 matrix of collision possibilities. In this experiment we examined observers' judgments of mass ratio as a function of which cell the collision event was embedded.

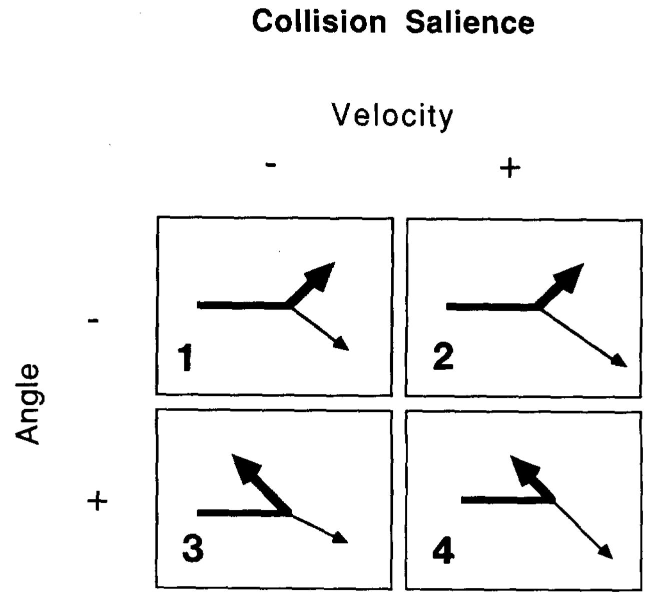

Figure 4. The basic collision structures in the 2 × 2 salience matrix. Both angle and velocity information have two levels of salience. Collisions with high angle salience were defined by the incoming ball ricocheting backward. Collisions with low angle salience had both balls scattering forward at acute angles. High velocity salience was defined by an exit velocity ratio in excess of 2. Low velocity salience was defined by exit velocity ratios less than 1.5. The incoming ball is depicted by the thicker line, and velocity magnitudes are represented by the length of the exit vectors. This 2 × 2 matrix defines the four categories of collisions that are discussed in the text.

Method

Subjects. Twelve University of Virginia students (6 male, 6 female) participated in this experiment for credit in an introductory course in psychology. All observers were naive as to the purpose of the study and had not participated in previous experiments or pilot studies.

Stimuli. Two levels of salience were discriminated in this experiment: high and low. High angle salience was defined in terms of ricochet of the incoming ball. In the low-angle-salience stimuli, the exit angles for both balls were less than 45°. In all cases the exit velocity of m_2, the ball which was struck, was greater than or equal to the exit velocity of the incoming ball, m_1. High velocity salience was defined in terms of whether the ratio v_{2f}/v_{1f} exceeded 2.0. In the low-velocity-salience stimuli this ratio was less than 1.5. Three collision events were chosen from each of the following four categories.

Category 1 = low velocity salience, low angle salience;
Category 2 = high velocity salience, low angle salience;
Category 3 = low velocity salience, high angle salience;
Category 4 = high velocity salience, high angle salience.

The stimuli, grouped according to category, are depicted in Figure 5. The salience of velocity and angle information is determined by how far from the axes in this space the collision event lies. Stimuli close to the origin have low salience in both angle and velocity. Stimuli close to one axis but distant from the second are salient on one dimension. Stimuli in the upper portions of the quadrant ($\log(v_2/v_1), \log(\alpha_1/\alpha_2) > 0$) lying close to the bisector are salient on both dimensions.

The stimuli in Category 1 could have been chosen either from the quadrant with $\alpha_1 > \alpha_2$, or from the quadrant with $\alpha_1 < \alpha_2$. In the former case this would have necessitated simulating collisions with a mass ratio near unity. Such a choice would have introduced the confound that if subjects could not discriminate mass ratio, the reason could be that none of the information was salient or, more likely, that the masses were not that different. By sampling collisions for Category 1 from the $\alpha_1 < \alpha_2$ quadrant, we were able to avoid this confound by choosing collisions that had large mass ratios. The mass ratios selected were identical to those selected for Category 2.

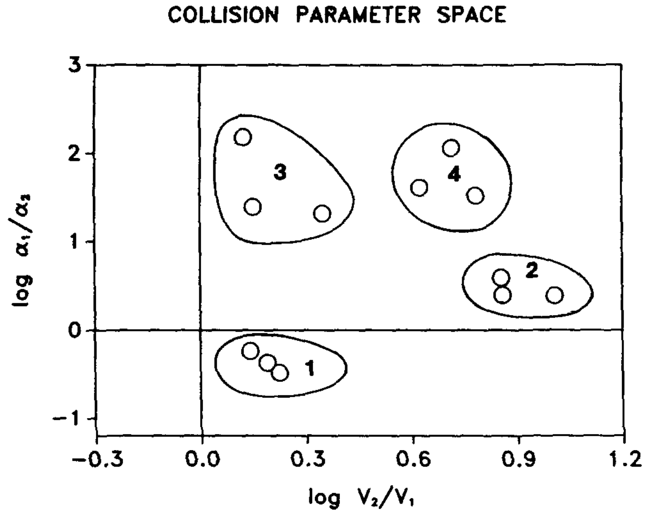

Figure 5. The collision parameter space as defined by the exit velocity ratios and exit angle ratios. Shown are the 12 collisions displayed in Experiment 2. Three collisions were selected from each cell of the salience matrix. The collisions groups are labeled by the category they represent. Collisions near one axis and far from the other are salient along a single dimension of information. Collisions far from both axes are salient on both dimensions of information, and collisions near the origin have no salient information. (α_1 = angle of exit of Ball 1, α_2 = angle of exit of Ball 2, v_2 = outgoing velocity of Ball 2, v_1 = outgoing velocity of Ball 1.)

The stimulus presentation was identical to that in Experiment 1, with two exceptions. In this experiment there was no occluded condition and subjects saw the entire event including the motion of the incoming ball. Second, the incoming ball always had a speed of 6.7 cm/s. The parameters for the 12 collisions displayed in Experiment 2 are shown in Table 3.

Design. A within-subject factorial design was employed with mass ratio and up–down presentation as factors.

Procedure. The procedure was identical to that of Experiment 1, except that in addition to judging which of the two balls was heavier, subjects in this experiment also indicated how dissimilar in mass the two balls appeared. The observer indicated which ball was perceived to be the heavier *and by how much*, by the differential placement of a cursor along a scale. Twelve practice trials were viewed that were representative of the collisions in the experimental trials but were not identical to them.

Table 3
Collision Parameters for Experiment 2

	Salience						
Category	Velocity	Angle	m_1/m_2	α_1	α_2	v_{1f}	v_{2f}
1	−	−	1.40	36	45	7.2	8.3
1	−	−	1.65	31	45	7.3	8.8
1	−	−	1.90	28	45	7.4	9.3
2	+	−	1.40	45	25	4.5	10.6
2	+	−	1.65	37	25	4.8	11.3
2	+	−	1.90	30	20	5.7	11.3
3	−	+	0.40	133	15	4.9	5.5
3	−	+	0.50	101	25	5.2	6.0
3	−	+	0.60	93	25	4.8	6.8
4	+	+	0.60	118	15	3.5	7.2
4	+	+	0.65	100	20	4.0	7.4
4	+	+	0.75	91	20	3.7	8.1

Note. All velocities are given in units where v_{1i} = 10. Angles are in degrees. m_1 = mass of Ball 1, m_2 = mass of Ball 2, α_1 = angle of exit of Ball 1, α_2 = angle of exit of Ball 2, v_{1f} = outgoing velocity of Ball 1, v_{2f} = outgoing velocity of Ball 2.

Results and Discussion

Two independent judgments were obtained from observers in this experiment. In addition to judging which of the two balls was heavier as a binary decision, observers also indicated how dissimilar in mass the two balls appeared. The results for these two tasks were analyzed separately. We shall discuss the results for the binary decision task first.

Analysis of binary decision. The overall ability to distinguish which of the two balls was heavier was analyzed in terms of the category to which the collision belonged. The 2 × 2 performance matrix is shown in Figure 6. Contrasts were computed that tested our heuristic theory of performance. In the theory, performance is determined by the presence of salient information that is coupled to a correct heuristic. The correct heuristics for the four categories were as follows:

Category 1: Velocity heuristic ($m_1 > m_2$). The lighter ball was slightly faster and scattered at a slightly greater angle.

Category 2: Velocity heuristic ($m_1 > m_2$). The lighter ball was much faster, but the heavier ball scattered at a slightly greater angle.

Category 3: Angle heuristic ($m_1 < m_2$). The lighter ball was slightly slower but ricocheted backwards.

Category 4: Angle heuristic ($m_1 < m_2$). The lighter ball was much slower but ricocheted backwards.

Observers' ability to judge which ball was heavier was consistent with a salience-based heuristic theory. Categories 2

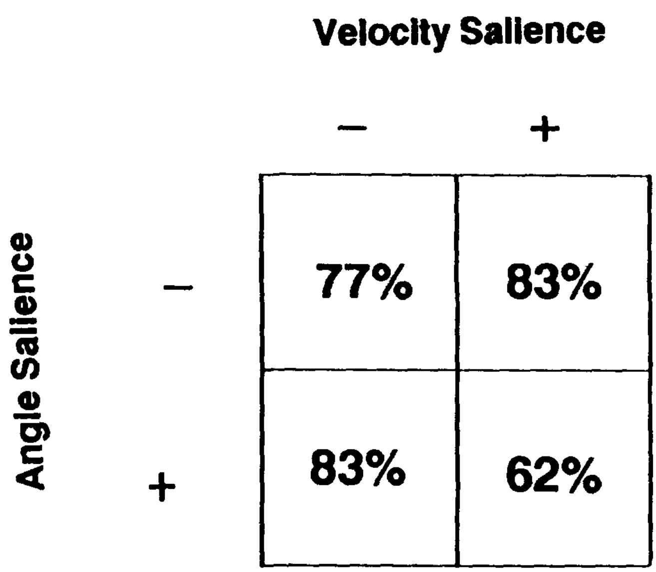

Figure 6. Results for Experiment 2. The percentage correct of observers' judgments of which of the two balls was heavier as a function of which salience cell the collision was drawn from. Performance is best when a single dimension of information is salient and worst when both dimensions of information are salient.

and 3 are cells for which there was a single category of salient information that was associated with a heuristic that happened to be true. Contrast analysis showed that performance in Categories 2 and 3 was indistinguishable, $F(1, 11) = 0.01$, $p < 0.93$. Collisions in Category 4 are singular for having contained two categories of salient information, only one of which was associated with a true heuristic. Performance in Category 4 was reliably worse than performance in cells containing a single category of salient information, $F(1, 11) = 34.25$, $p < 0.0001$. The poorer performance found in this cell was due to subjects' being deceived by the rapid exit velocity of the heavier ball. Collisions in Category 1, conversely, contained information of low salience, although all of the information was associated with heuristics that happened to be true; that is, the lighter ball both was faster and scattered at a larger angle. The performance in Category 1 collisions was marginally lower than performance in cells containing a single category of salient information, $F(1, 11) = 2.92$, $p < 0.088$.

Comparison with Todd and Warren (1982). Our results for the binary decision problem are consistent with those found by Todd and Warren (1982) in the condition they tested for elastic collisions between two objects (represented as squares), one initially stationary. In their one-dimensional collision experiment there were also two categories of information based on a speed and angle comparison, although all of the information was not always present. When $m_1 > m_2$ (incoming ball heavier), both balls scatter forward and the lighter ball is faster. This condition is similar to our Category 2, and observers were virtually perfect given that the only category of information that one could base a judgment on was speed, and this was coupled to a correct heuristic. However, when $m_1 < m_2$ (incoming ball lighter), the incoming ball scatters backward while the stationary ball scatters forward. In this regime, angle information was also available as a source for judgment. Here Todd and Warren found that subjects performed at chance or worse when m_1 was only slightly less than m_2 and that performance monotonically improved as m_1/m_2 approached zero. In order to evaluate these results it is necessary to understand how the events appeared to observers.

The mass–velocity relation derived from the energy and momentum conservation equations is required for relating Todd and Warren's results to the present results. This relation is $v_2/v_1 = 2/(1 - m_2/m_1)$. When m_2 is only slightly greater than m_1, say $m_2/m_1 = 1 + e$, where e is a small number, then $v_2/v_1 = -2/e$. Here we see that m_1 ricochets backward (note the minus sign), but that the heavier ball, m_2, has a much greater velocity. This situation corresponds to our Category 4 collisions where both angle and velocity information were salient, and observers were brought into conflict over which heuristic to base their judgment on; both balls satisfied a lightness criterion. Now as m_2/m_1 approaches 3, the velocity ratio v_2/v_1 approaches unity monotonically. Thus, in the neighborhood of $m_1/m_2 = 0.33$, we recover the set of Category 3 collisions where angle salience was high and velocity information was suppressed. Indeed Todd and Warren found that their subjects were 90% correct at $m_1/m_2 = 0.33$ ($v_2/v_1 = 1$), 75% correct at $m_1/m_2 = 0.50$ ($v_2/v_1 = 2$), but only 48% correct at $m_1/m_2 = 0.67$ ($v_2/v_1 = 4$). The performance functions derived from Todd and Warren's study can in fact be viewed as a psychophysical calibration of the relative salience of velocity ratios in the presence of ricochet.[1]

The observers in Todd and Warren's experiments apparently were making judgments based on the salience of the two categories of information present in the postcollision epoch. It is noteworthy that Todd and Warren's data can be accounted for in terms of postcollision epoch information even though this information is, in principle, insufficient for accurate mass ratio judgments in one dimension. Their observers were using heuristics appropriate to the general class of two-dimensional collisions even when restricted to observing the one-dimensional family.

The primary difference between our approach and that of Todd and Warren is that we do not attempt to account for the data with a single heuristic. Todd and Warren were unable to find a single heuristic that could simultaneously predict good performance when $m_1 > m_2$, the performance discontinuity for m_1 slightly less than m_2, and the improved performance as $m_1/m_2 \rightarrow 0$. We do not believe that these data can be explained in terms of a single heuristic. Instead, chance performance is explained in terms of the competition between two heuristics that are defined on different dimensions of information. Similarly, good performance is expected only when one dimension of information is highlighted and one dimension is suppressed.

Analysis of perceived dissimilarity. The critical test for a theory of judgment based on salience is provided by the distributions of perceived dissimilarity. Independent of the veridicality of the judgment of perceived mass ratio, a salience theory predicts that perceived mass ratio is proportionally related to perceived salience, not to the distal mass ratio. These distributions are shown in Figure 7, which shows the frequency tables for judged dissimilarity in mass. For this analysis the dissimilarity scale was divided into 10 equal intervals. A score of 1 indicated m_1 was only slightly greater than m_2. A score of 5 indicated that m_1 was much greater than m_2. Conversely, a score of -1 indicated that m_2 was slightly greater than m_1, and a score of -5 indicated that m_2 was much greater than m_1. A bin corresponding to a score of 0 was included in order to display the frequency of judgments of exact equality.

The expected distributions based on a salience theory are (a) skew distributions when a single category of information was salient, (b) bimodal distributions when two categories of

[1] A degree of freedom not explored in the present experiments, but contemplated by Runeson (1977) and empirically studied by Todd and Warren (1982), is the role of elasticity. In the latter studies it was generally found that the ability to judge mass ratio declined with decreasing elasticity. This pattern of results is interpretable in terms of the reduction in salience of the critical information that people use in accessing heuristics. As elasticity decreases, ricochet is suppressed and the ratio of exit velocities approaches unity. Highly inelastic objects, such as clay, do not ricochet for any value of impact parameter. When the salience of the available information is sufficiently reduced, observers are apparently unable to employ the heuristics that generally lead to accurate judgments.

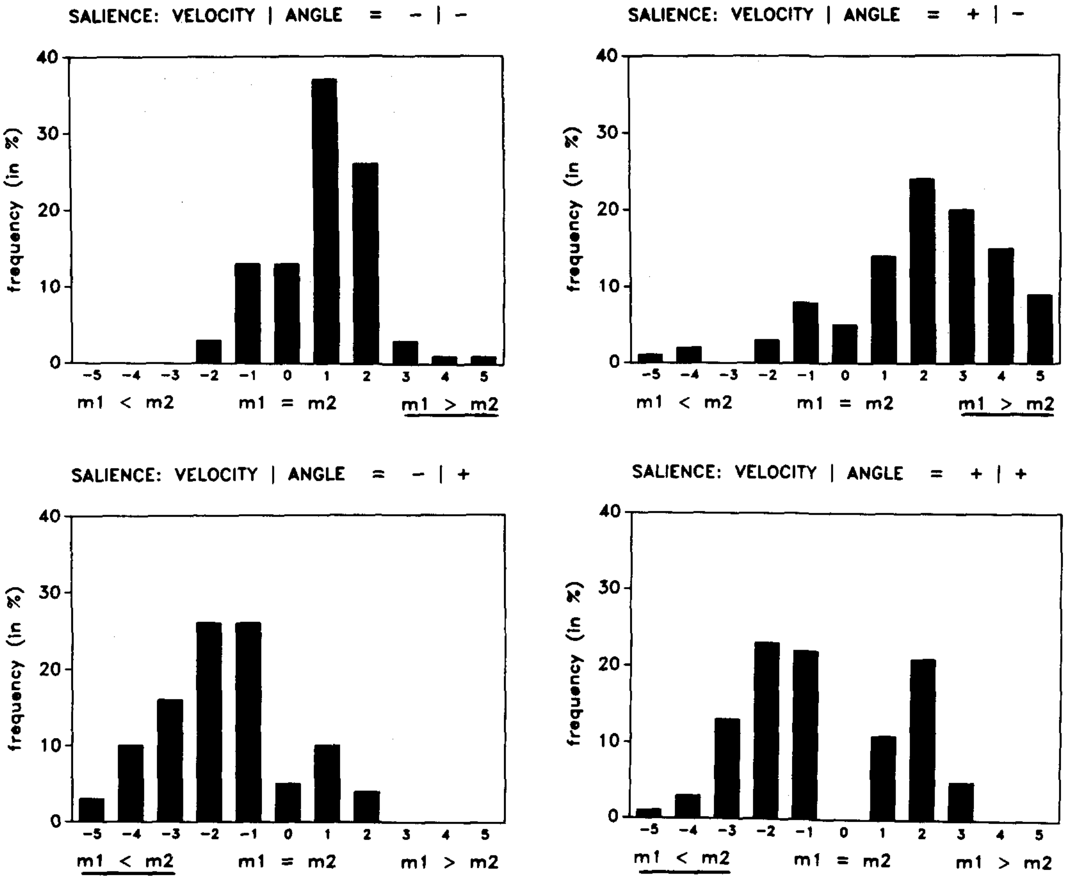

Figure 7. Results for Experiment 2. Shown are the frequency distributions of perceived dissimilarity in mass as function of salience. The correct response is indicated by which of ($m_1 < m_2$) or ($m_1 > m_2$) is underlined. In the cells described by one salient dimension the distributions are skewed away from perceived mass equality. When both dimensions are salient the distribution is bimodal, with both populations skewed from perceived mass equality. When no information is salient the distributions are displaced toward perceived mass equality. (m_1 = mass of Ball 1, m_2 = mass of Ball 2.)

information were salient, and (c) a centrally peaked distribution only slightly displaced from mass equality when no category of information was salient. These features are clearly evident in the 2 × 2 matrix of perceived mass ratio in Figure 7. Both Categories 2 and 3 (one category of salient information) showed skew distributions. The mean ratings for these distributions were 2.1 and −1.7, respectively, for Categories 2 and 3. The distribution in Category 1 (low salience information, mean rating = 1.2) was not displaced far from perceived mass equality. Most important, the distribution for Category 4 (two categories of salient information) was bimodal. Observers in this condition were internally consistent, and the bimodality was due entirely to disagreement between individuals. (On the four viewings that each of 12 observers had of the three collisions in Category 4, observers contradicted themselves twice on 3 occasions, once on 11 occasions, and not at all on 22 occasions.) When two categories of salient information were present, observers were divided about which ball was heavier, but they were unanimous that the balls had quite dissimilar masses. In this cell alone there were no ratings of mass equality.

Two sample two-tailed Kolmogorov-Smirnov tests were run on all pairs of distributions shown in Figure 7 in order to quantify the degree of difference that existed between them. These results are shown in Table 4. In order to obtain a meaningful comparison across the conditions where $m_1 > m_2$ was correct and where $m_1 < m_2$ was correct, the distributions for Categories 3 and 4 were mirror reflected across the central

Table 4
Kolmogorov-Smirnov Analysis of Mass-Ratio Ratings

Comparison	D_{max}[a]	Probability[b]
Category 1 vs. Category 2	0.41	<.0001
Category 1 vs. Category 3	0.24	<.0003
Category 1 vs. Category 4	0.28	<.0001
Category 2 vs. Category 3	0.19	<.008
Category 2 vs. Category 4	0.28	<.0001
Category 3 vs. Category 4	0.27	<.0001

[a] Maximum difference between the cumulative probability distributions generated from the frequency tables.
[b] Probability that a D_{max} or larger would be obtained under the null hypothesis that the samples are drawn from the same distribution.

point of mass equality; that is, all correct responses were placed on the right. All the distributions were significantly dissimilar. (The number of degrees of freedom in all tests was 144.) In a Kolmogorov-Smirnov test, the maximum difference between the distributions is scaled to be less than or equal to unity. The differences found between Categories 1 and 2 and between Category 4 and all others exceeded 0.25, which in this context is quite large.

The rank ordering of the maximum difference between the integrated frequency distributions, D_{max}, was essentially given by the relative positions of the various clusters of responses. Categories 2 and 3 had the smallest value of D_{max} because they differed only in the extent to which their clusters were skewed from mass equality. Categories 1 and 2 were the most dissimilar on this test because they were both single-cluster distributions and the cluster in Category 1 converged on mass equality while the cluster in Category 2 was the most highly skewed. This result shows clearly that perceived dissimilarity in mass was governed by salience and not by the distal mass ratios because the mass ratios in Categories 1 and 2 were identical. The large differences between Category 4 and all others were generated by the presence of a large cluster of incorrect responses corresponding to the fact that roughly half the observers were deceived by salient but irrelevant velocity information.

The interpretation of these results is straightforward. If an incoming ball strikes a stationary ball and causes that ball to move away at high velocity, the struck ball looks a good deal lighter. Similarly, if the incoming ball ricochets backward, the incoming ball looks much lighter. If both features are present, regardless of which feature forms the basis of judgment, one of the balls looks a lot lighter. Observers simply disagree about which feature to base a judgment on when both features are salient. If neither feature is salient, the incoming ball does not ricochet, and the struck ball is not that much faster, then the masses do not appear to be dissimilar even if, in fact, they are.

General Discussion

The present experiments surveyed the abilities that people have in the judgment of mass ratio over a large region of the parameter space defining natural collisions. Experiment 1 showed that the information available in the precollision epoch is not required. In Experiment 2 we tested a formal heuristic model. It was shown that people make mass ratio judgments on the basis of two heuristics—ricocheting balls appear lighter, and faster moving balls appear lighter. By pitting these two heuristics against each other we demonstrated that angle information is isolated from velocity information and that access to velocity and angle heuristics is controlled by the differential salience that these categories of information have for a given individual. This model provides an account not only for our data but also for the one-dimensional collision data reported by Todd and Warren (1982).

The studies reported here focused on dynamic understandings of collisions, but they have implications for the general class of KSD theories that claim that accurate dynamic judgments are possible when the available kinematic information is sufficient. An important lesson that is illustrated by these experiments is that one cannot generalize about human abilities for a task until performance has been measured over a large region of the parameter space that defines that task. In particular, it is not clear what limitations exist in the perception of lifted weight. The fact that under some circumstances people can estimate weights lifted by actors does not mean that this is a general ability. Unlike collisions, the information that people use in making such judgments has never been specified, and consequently it is not clear what constraints have been imposed in the experimental designs. The critical experiments that are required for determining the recovery of dynamic properties from kinematic data cannot be performed until a characterization of these data is available.

The fundamental difference between KSD theory and the model of event perception developed by Proffitt and Gilden (1989) and amplified in the present heuristic theory is in the way people are supposed to process information. The central issue is how people relate different categories of information. Collisions provide a unique opportunity for addressing this issue because one can give an explicit description of the event at a kinematic level and can control the event with great precision.

Our results show that for collisions, angle and velocity information is not transformed into a multidimensional quantity as required by classical mechanics, but is related only through a salience function. The bimodality found in Category 4 collisions confirms that observers split the information up; they do not form a compromise or trade-off when confronted with two conflicting categories of information. A physical theory of collisions, however, is formulated in terms of multidimensional quantities. The appropriate quantities for the determination of mass ratio are the projections of the momentum vectors onto orthogonal axes. Observers do not derive these projections; instead they treat the magnitude of the velocities along the trajectories and the angle of the trajectories as separate categories of information.

There are domains outside of motion events where it has been demonstrated that people do not form synthetic multidimensional quantities when confronted with stimuli that vary simultaneously on several dimensions of information. The decision problem for determining mass ratio in collisions is reminiscent of studies conducted by Shepard (1964a, 1964 b) in which subjects were shown circles of different sizes that had radial lines drawn at different compass points. Circles could be judged to be similar on the basis of size or line orientation. As in the collision event, there is no clear way to relate line orientation and circle size into a single multidimensional quantity that would allow comparison on both dimensions simultaneously. Shepard found that subjects formed a bimodal distribution when they made a similarity judgment on an ensemble of stimuli that varied simultaneously in circle size and tick orientation. A compromise that could have been effected by taking both into account (say, by placing the sizes and orientations into a metric space and computing a minimum distance) was not done. Shepard (1964a, 1964b) proposed quite generally that people treat decision problems involving several dimensions of information as if the problems were unidimensional.

There are a variety of circumstances in which people are required to form judgments about events or situations that have a high dimensionality. The ways in which information is processed in these environments are central to understanding the abilities that people have in their general dealings with the world. Studies such as those reported here and by Proffitt and Gilden (1989) suggest that people cope with multidimensionality by creating fictions (heuristics) that allow them to handle several categories of information without having to figure out how they go together.

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Appendix

Physical Analysis of a Two-Body Collision

A collision is determined by specifying the ball diameters, D, the impact parameter, s, and velocity of the incoming ball, v_{1i}. Conservation equations determine the exit speeds, v_{1f} and v_{2f}, and the exit angle of the incoming ball, α_1. The exit angle of the struck ball, α_2, is determined from a geometric boundary condition. Mass of Ball 1 and mass of Ball 2 cannot be calculated independently in a collision. They can only be determined up to their ratio, which we have defined as Q.

The conservation equations for this system are as follows:

$$x\text{-momentum: } Q\, v_{1i} = Q\, v_{1f} \cos \alpha_1 + v_{2f} \cos \alpha_2. \quad \text{(A-1)}$$

$$y\text{-momentum: } Q\, v_{1f} \sin \alpha_1 = v_{2f} \sin \alpha_2. \quad \text{(A-2)}$$

$$\text{energy: } Q\, (v_{1i})^2 = Q\, (v_{1f})^2 + (v_{2f})^2. \quad \text{(A-3)}$$

In the limit of hard-sphere scattering, Ball 2 exits on a line connecting the centers of the two balls at the moment of impact. In terms of the geometric properties of the balls, we have

$$\sin \alpha_2 = s/D. \quad \text{(A-4)}$$

The determination of Q, the mass ratio, is possible from any of the three conservation equations given the speeds and angles available in the distal event. The following three independent expressions all suffice for computing Q:

$$Q = v_{2f} \cos \alpha_2/(v_{1i} - v_{1f} \cos \alpha_1) \text{ from } x\text{-momentum.} \quad \text{(A-5)}$$

$$Q = v_{2f} \sin \alpha_2/(v_{1f} \sin \alpha_1) \text{ from } y\text{-momentum.} \quad \text{(A-6)}$$

$$Q = (v_{2f})^2/((v_{1i})^2 - (v_{1f})^2) \text{ from energy.} \quad \text{(A-7)}$$

The equations for the one-dimensional case of head-on collisions are given by restricting $\alpha_2 = 0$, $\alpha_1 = 0$, 180°. This restricted set formed the basis for Runeson's (1977) and Todd and Warren's (1982) analyses.

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