

# Cognitive Emissions of 1/f Noise

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The residual fluctuations that naturally arise in experimental inquiry are analyzed in terms of their time histories. Although these fluctuations are generally relegated to a statistical purgatory known as unexplained variance, this article shows that they may harbor a long-term memory process known as 1/f noise. This type of noise has been encountered in a number of biological and physical systems and is theorized to be a signature of dynamic complexity. Its presence in psychological data appears to be associated with the most elementary aspect of cognitive process, the formation of representations.

The work described here concerns memory and the temporal evolution of cognitive activity. Although many of the ideas presented make little contact with current cognitive theories, almost all of the empirical work derives from the well-known observation that memory inevitably makes an appearance in repeated episodes of measurement. Explicit memory, for example, was a bedeviling factor in the early studies of magnitude estimation, signal detection, and absolute identification. People have a tendency to repeat themselves so that if they have just said “loud,” they are likely to say “loud” again (Luce, Nosofsky, Green, & Smith, 1982; Stadon, King, & Lockhead, 1980), or if they have just said “yes,” another “yes” is likely to follow (Verplanck & Blough, 1958; Verplanck, Collier, & Cotton, 1952; Verplanck, Cotton, & Collier, 1953). More important in modern applications are the implicit associations that lead to hysteresis and sequential priming. Were it not for the threat of systematic bias, threshold measurement, for example, would not require interleaved staircases or randomization of stimulus presentation. The use of Latin squares in the analysis of variance has the same motivation—the avoidance of order effects. The implicit understanding that underlies all efforts to minimize the role of memory in repeated measurement is that sequential correlations are not relevant to the particular construct being measured—say an auditory or visual threshold. Deciding what is or is not relevant in psychology is a very subtle undertaking and at least demands that one knows what causes sequential correlations. In this article, I show that there is considerably more to these correlations than has been presumed. Sequential correlation is not only an intrinsic part of psychological measurement; its structure may provide new perspectives on the mechanisms of thought that lead to discrimination and choice.

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The methods used here are primitive and consist solely of constructing the consequences of introducing time as an essential component of measurement. Standard methods of assessment in cognitive psychology are, in contrast, state-based as opposed to time-based. What this means in practice is that data deriving from a single observer are generally not kept intact as a response history but are diced up among treatment cells that express the design. In a successful experiment, different treatment cells come to be associated with different mental states. How mental states stochastically evolve in time is generally not an issue, but here it is the single focus of inquiry. As a consequence, in all of the work described here, I either dispense with stimuli altogether, present a single stimulus in the course of an entire experiment, or subtract from the data the effects attributable to treatments. In this way, I obtain signals that have no conceivable relevance to how psychological states are differentiated but that do contain all of the temporal correlations that are induced by choice and discrimination behavior.

## Fluctuations in Speeded Response

The constellation of ideas that provides the setting for this work is well outside of experimental psychology. Most of the relevant articles and congresses derive from statistical physics, solid state physics, and biophysics. However, the principal findings are not esoteric and have immediate application to both psychological theory and practice. So I begin with a series of demonstrations in speeded response that make this point, and the best way to do this is to work through a concrete example in some detail. In order for this example to be effective, it should be one that is familiar, has been replicated by many independent investigators, and has as solid a theoretical foundation as might be hoped for. Mental rotation, a paradigm developed by Shepard and coworkers (Shepard & Cooper, 1982), satisfies these requirements admirably. In Figure 1, the data from a mental rotation experiment (Gilden, 1997) are presented (six observers; 1,056 trials each; judgments made on the mirror inversion of *R*, *P*, or *F* at angles of 0°, 60°, 120°, 240°, and 300°; trials were self-paced with no feedback). The data shown in Panel A resemble those typically presented in textbooks and in a myriad of research articles on this subject. The asymmetry between 120° and 240° even has an interpretation in terms of the consistent rightward frame of the character set. These data surely create the impression that judgments of mirror reflec-

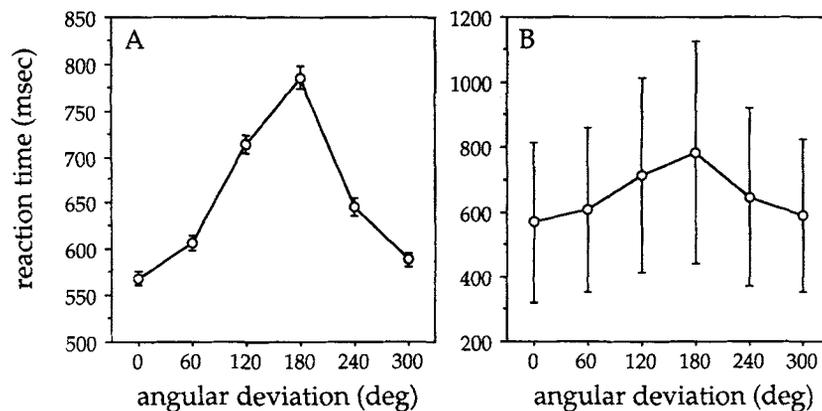


Figure 1. Results from a typical study of mental rotation. Letters appeared at various angles, and observers made speeded judgments about whether they were mirror-inverted or not. Panel A shows how the error is usually presented in the literature in terms of standard errors of the mean. Panel B shows error bars that depict the standard deviation of the raw data.

tion are dominated by angular deviation and motivate the interpretation that visual imagery has spatial properties, and so forth. This impression, however, is not entirely faithful to the data. The error bars related to reaction time measurement reported in textbooks (not generally defined) and in research articles (almost always defined) are standard errors of the mean and so reveal much about the degree to which the mean can be localized and quite little about the true variability in the data. Although this observation may appear truistic, it underscores an important property of reaction time data. Our ability to localize the mean does not entail that the treatment effects explain the data (the treatments here are the different angles of presentation). Panel B shows the same data plotted with error bars, which are standard deviations of the reaction time distributions at each angle. This way of looking at the data reveals that the angle variable has only a small influence on the time required to make a judgment of letter inversion, and in fact treatment effects in this experiment account for only about 10% of the total reaction time variance within individual observers. In a sense, 90% of what observers give in the way of reaction time data has nothing to do with mental rotation. It is this 90% that I am interested in here.<sup>1</sup>

#### Latent Structure in Reaction Time Histories

Although it is clear that most of the data received in the mental rotation experiment cannot be used to understand the mapping between response latency and letter orientation, there is no reason to suppose that there are not other kinds of structure latent in the sea of fluctuations. To explore this possibility, I treat the data as an intact history of trials. Figure 2 displays the reaction times for a single observer and their decomposition into means and residuals. The top sequence shows the raw response latencies in the exact order that they were produced by the observer. The middle sequence is a fluctuating signal over 12 discrete values that is simply a record of the cell means corresponding to a random counterbalanced presentation of stimuli (six angular deviations, two levels corresponding to whether the letter was mirror-inverted or not). The bottom sequence is the history of residual fluctuations formed as a difference between the raw and cell mean sequences. The fact

that the raw data and the residuals look alike is due to the small influence of the independent variables on responding. More important, the raw data, and consequently the residual fluctuations, have little waves running through them. These little waves are visual evidence of positive correlation. Reaction time latencies have an imperfect but manifest tendency to replicate themselves. Such waves are not seen in the record of the cell means because this sequence is explicitly counterbalanced and randomized. The waves in the residuals have a structure that is reminiscent of random fractals—a nested structure within a wide range of scale, where scale in this context is indexed by trial number. Where does this structure come from? It must arise from memory processes within the observer. Were there no memory, the residuals would resemble the sequence of cell means at larger amplitude.

In order to understand the kind of structure that these waves represent, it is necessary to introduce the more general context of correlated noises. The central issue is that noises or fluctuations come in a variety of forms; there is not just one thing called "noise." It is the case that the important noises in physics, biology, and psychology are members of a single family that is parameterized by the internal correlations between successive increments. The noises in this family are referred to as *fractional Brownian motions* and are most easily described in terms of their power spectra. Fractional Brownian motions have power-law power spectra, power  $\approx 1/f^\alpha$ . In the log-power/log-frequency plane, the spectra are simply straight lines with slope  $-\alpha$ .  $\alpha = 0$  corresponds to no correlation between successive increments, whereas noises with positive sequential dependencies have  $\alpha > 0$  (for a discussion of the transformational properties of this family, see Gilden, Schmuckler, & Clayton, 1993).

Three especially important noises are illustrated in Figure 3 with their associated power spectra. The top panel displays an example of white noise. White noise has a flat power spectrum reflecting

<sup>1</sup> The variability in reaction time attributable to readout from the key-press makes a negligible contribution. The standard deviation of the readout error is 4 msec, a value that is much smaller than any feature of interest in this article.

the lack of correlation between increments. This type of noise is generally assumed to characterize the fluctuations in the collection of data. Recall the first and most fundamental equation of statistics:

$$\text{observation} = \text{cell mean} + \text{error}. \quad (1)$$

Analysis of variance (ANOVA) assumes that the observations collected in the course of an experiment have independent error terms (i.e., they are random and independent samples from some distribution). At face value this is not a bad assumption, although it is by now clear that it is violated in data deriving from a single individual. White noise is the garden variety encountered most generally in natural systems. The fluctuations observed in the cell means in Figure 2 are a white noise because the random interleaving of trials is guaranteed to produce random and independent samples.

Another commonly encountered noise is illustrated in the lower panel. Brown noise is called such because of its relation to Brownian motion, the path that particles execute as they diffuse in a random walk. Brown noise can be formally constructed by computing a running sum over the increments of a white noise. At any moment, the running sum gives the current position of a random walk. Random walks are highly self-correlated because successive positions have an entire history in common and differ by only a single displacement. Positive self-correlation is manifested in random contour by the appearance of slowly undulating hills and valleys that support jagged high-frequency structures (see Gildea et al., 1993, for a discussion of how people perceive Brownian motions). It is not happenstance that landscape terminology seems apt in the description of random-walk contour. Natural landforms are typically random walks (Burrough, 1981; Keller, Crownover, & Chen, 1987; Sayles & Thomas, 1978a, 1978b; van der Schaaf & van Hateren, 1996; Voss, 1985, 1988). The spectral signature of this kind of structure is a rapid drop-off in power with frequency; low frequencies (large scales) have large amplitude whereas high frequencies (small scales) have low amplitude. Random walks have power spectra that fall off precisely as  $1/f^2$ .

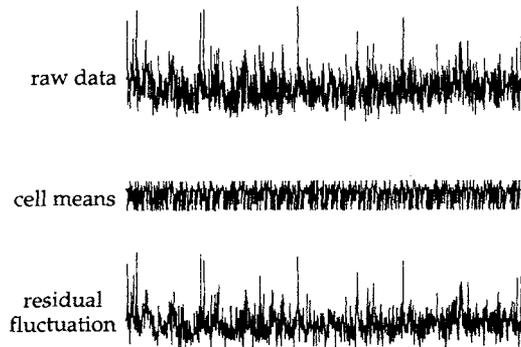


Figure 2. Raw mental rotation reaction times from a single observer. The data are plotted in the trial order in which they were collected. The top series shows the raw data, the middle series shows the cell mean for each particular stimulus (defined by angular orientation and mirror reflection) in the order in which it was shown, and the bottom row shows the trial-ordered residuals.

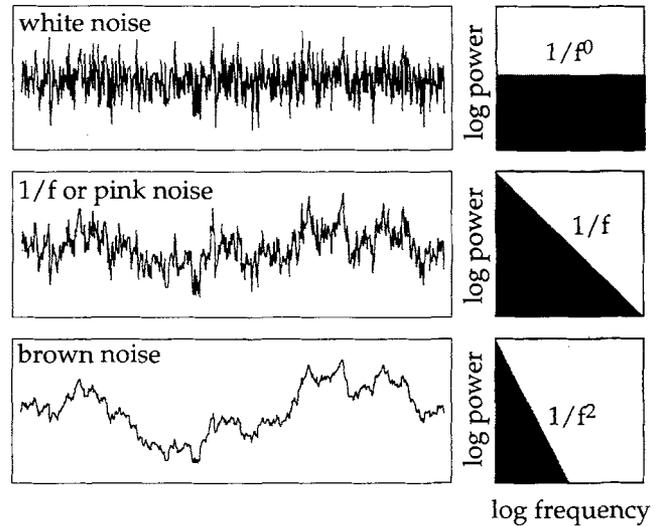


Figure 3. The three canonical types of fractional Brownian motion. Examples of each motion are shown along with its power spectrum. These motions are self-affine fractals; frequency scaling can be offset by amplitude scaling.

Sandwiched exactly between noises with random increments (white noise) and noises that are sums over random increments (brown noises) is a type of noise that has particular physical significance. This noise is referred to variously as  $1/f$  (the power of such noises varies inversely with frequency), flicker noise, and pink noise.<sup>2</sup>  $1/f$  noise is illustrated in the middle panel of Figure 3. In the past quarter century,  $1/f$  noise has been discovered in the temporal fluctuations of an extraordinarily diverse number of physical and biological systems (Press, 1978; see articles in Handel & Chung, 1993, and references therein). Examples of its occurrence include fluctuations in tide and river heights, quasar light emissions, heart beat, firings of single neurons, and resistivity in solid state devices.

The most accessible introduction to the significance of  $1/f$  noise is one given by Martin Gardner (Gardner, 1978) in his *Scientific American* column "Mathematical Games." In this particular column, Gardner asked for the sense in which music imitates nature. Sounds in nature are not musical in that they tend to be either too repetitive (bird song, insect noises) or too chaotic (ocean surf, wind in trees, and so forth). The answer to this question was given in a statistical sense by Voss and Clarke (1975, 1978), who showed that pitch and loudness fluctuations in speech and music are  $1/f$  noises. So music is like tides not in terms of how tides sound, but in how tide heights vary. In expounding this result, Gardner pointed out that the sense we have that music is interesting and enjoyable to listen to is in large part due to its statistical correlations. Gardner considered three modes of piano performance to illustrate this point. In the first mode, the performer hits keys at random. This produces a white noise (flat spectrum) that is diffi-

<sup>2</sup> The vision community has recently taken to referring to landscapes as  $1/f$  noises because they prefer to make reference to the amplitude spectrum rather than to the power spectrum. The power spectrum is the square of the amplitude spectrum.

cult to listen to because notes can never be anticipated. White melodies are characterized by continuous surprise. The second mode is to execute a random walk on the keyboard. The rule for this mode is start at a random note and move to the left or right by one note with equal probability. Here the increments are uncorrelated, but the absolute position of the current note is highly correlated with the past. Such a melody is a brown noise (spectrum decaying as the square of frequency), and it is difficult to listen to because each note can be perfectly predicted within a three-note window. Brown melodies suffer from too much anticipation. What is interesting, musically speaking, are sequences that are neither too predictable nor too chaotic. From a purely statistical point of view, the noise exactly between these two modes is  $1/f$  noise, and this is what music is at the level of two-point correlation.

The connection between  $1/f$  noise and music provides a heuristic for understanding what kind of thing  $1/f$  noise is: It is the statistical embodiment of the synthesis between disordered high information activity (white noise) and highly ordered low information activity (random walk noise). This synthesis, although intuitive and easy to state, turns out to be quite difficult to realize in statistical or physical models of nature.

### Two Sources of Fluctuation in Speeded Response

With this background, the problem of identifying the types of noise produced by mental rotation may be meaningfully addressed. The first issue is to determine if the residual fluctuations are contained within the family of fractional Brownian motions. Answering this question involves reducing the wave structure that is visible to the eye in terms of its correlational structure. Spectral analysis provides exactly the tool required. The average power spectrum of mental rotation latency residuals is shown in Figure 4. Note that in this context the received data (reaction times) are

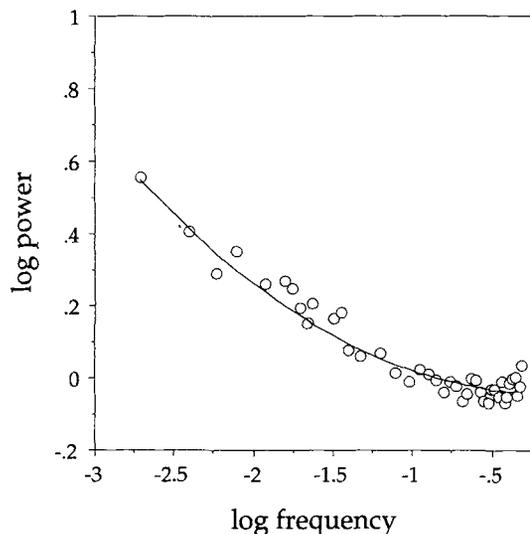


Figure 4. The power spectrum of mental rotation residuals together with the optimal fit from a two-source model that blends colored and white noise. Frequency here refers to inverse trial number. The axes have scales that are fixed up to the addition of a constant. Logarithms here and elsewhere are base 10.

indexed by the trial on which they occurred rather than, say, the time of their occurrence. For this reason, frequency here has the dimensions of cycles per trial rather than the more familiar unit of cycles per second (Hz). The spectral power is fixed up to an arbitrary normalization, which in the log-log plane means that the intercept will not be interpreted. The spectrum was computed using methods that provide the minimum variance at each estimated point (Press, Flannery, Teukolsky, & Vetterling, 1992). These techniques are described fully in Press et al. (1992) and in Gilden, Thornton, and Mallon (1995).

The first and most obvious feature of this spectrum is that the sequence of reaction time residuals does not conform to the assumptions that are generally made in interpreting reaction time data. The residuals are not independent samples drawn from any distribution. This is an important point and deserves emphasis. Regardless of the shape of the distribution of reaction times—ex-Gaussian, log normal, or whatever—random and independent samples will always produce a white noise. White noises have flat spectra, and this spectrum is not flat. More important than the rejection of residual independence is that the spectrum has an identifiable shape: It monotonically increases with decreasing frequency. Although these data are ordered by trial number, the interpretation that frequency receives is essentially the same as in applications where frequency is hertz or wave number (inverse units of distance). Low frequencies correspond to large blocks of trials and high frequencies to neighboring trials. The fact that the power increases at low frequencies means that there are waves of all scales running through the data, and that the waves with the largest amplitudes exist at the largest scales. This structure is produced by some kind of memory that persists over long periods of time (tens of minutes) and over hundreds of trials. This is an unanticipated result, and it suggests that there is an underlying coherence in residual structure.

The spectral representation of the mental rotation residuals provides clear and immediate evidence that the latency residuals are not a pure form of one of the members of the fractional Brownian motion family; the spectrum is not a straight line. As a first step toward understanding what this spectrum signifies, the residual fluctuations have been modeled in terms of a constrained mixture of two members of this family. The constraint is that one of the members be a pure white noise. The rationale for this constraint is that reaction time measurement integrates across both cognitive and motoric responses to a stimulus, and there is considerable evidence that timing fluctuations in keypress activation are truly independent (Wing & Kristofferson, 1973; Wing, 1980). That is, at least some of the variability in reaction time is a white noise. There may also be white fluctuations arising from the decisional and perceptual parts of the task, but it is clear that there is an active source of correlated fluctuations. The model of the reaction time residuals groups all sources of white variation together and represents the residual on the  $n^{\text{th}}$  trial as being embedded in an ordered sequence of the following form:

$$\text{residual}_n = (1/f^\alpha)_n + \beta N(0, 1), \quad (2)$$

where  $(1/f^\alpha)_n$  is the  $n^{\text{th}}$  term in a  $1/f^\alpha$  noise scaled to have zero mean and unit variance,  $N(0, 1)$  denotes a sample from the normal distribution with zero mean and unit variance, and  $\beta$  is a constant free parameter that determines the relative contributions of the two types of variability. For each choice of  $\alpha$  and  $\beta$ , there is a unique

spectrum, and it is a relatively straightforward matter to find the values that provide a best fit to the spectrum shown in Figure 4. The best fit is displayed as the solid curve in this figure, and it is defined by  $\alpha = .7$  and  $\beta = 2$ .

Two conclusions are entailed by the application of the two-source model to these data. First, the correlated component is claiming a substantial fraction of the residual variance, 20% to be exact. In this sense, 20% of the original unexplained variance has now received some definition. To put this number into perspective, recall that letter orientation accounted for only 10% of the total variance. The second point is that although the correlated component is not exactly 1/f, it is quite close and would fall within the purview of 1/f phenomenon in the biology and physics literature.

Finally, it should be understood that although this discussion has been framed in terms of latency residuals, the treatment effects in mental rotation are so marginal that the same spectra are obtained for the raw latencies. The signal that is emitted in a mental rotation experiment patently contains more information about the observer than it does about the stimuli. The conception of the perceiver as a noisy information channel is not falsified by these data, but it misses the crucial point that people add information in their responses, and this information comes about in the first place because the decision process is occurring within a nervous system.

### *A Class of Fluctuations*

The generality of these findings was explored in Gilden (1997) by examining fluctuations in response latency over three additional domains: lexical decision, serial visual search, and parallel visual search. The tasks and methodologies used in these studies clearly do not exhaust the practice of experimental psychology, but they do provide a sampling of the usage of speeded response in typical applications. The basic results are reviewed both for their rhetorical value in the present argument and as preparation for the usage of latency fluctuations in the Monte Carlo simulations below.

The search data described here were generated as part of continuing study into the perception of motion fields (Thornton & Gilden, 2000). We use a method of multiple target search (van der Heijden, La Heij, & Boer, 1983) that ideally has the power to distinguish a serial process from one that is parallel but of limited capacity (see Townsend, 1990, for a discussion of these issues). In the studies described here, set sizes were one, two, or four, and there could be as many targets as the set size permitted. In the rotation experiment, targets were clockwise rotating disks and distractors were counterclockwise rotating disks. In the translation experiment, targets were rightward moving gratings and distractors were leftward moving gratings. Six observers made speeded decisions as to the presence of at least one target over 1,152 trials. The pattern of means suggested that rotation sign is processed serially whereas translation sign is processed in parallel. The lexical decision task, on the other hand, was fabricated simply to generate residuals. The stimuli in this experiment were lists of five real words or pronounceable pseudowords (taken from Juola, Ward, & McNamara, 1982). The number of real words was either one, two, three, or four in each list, and the task for the six observers was to identify this number. There were 1,280 trials in this experiment. As in the mental rotation study, trials in search and lexical decision were self-paced, and no feedback was given. The average data

expressed in terms of means (upper panels) and power spectra of the residuals (lower panels) are shown in Figure 5.

These additional studies show that the residual structure found in the mental rotation study is not unique. The paradigms assembled here are diverse in terms of what the observers are thinking about, the set size of possible responses, the set size of possible stimuli, and how long it takes the observers to arrive at a decision. Yet there is a clear consistency in the correlational structure of the residual fluctuations. The two-source model, plotted as a curve in each lower panel, validates what is obvious to the eye: There is little difference in the model parameters required to fit the spectra (Table 1 in Gilden, 1997). The derived exponents all fall in the range [.7, .9]. The only notable difference was in the proportion of variance attributable to white noise. There is more white noise in lexical decision and mental rotation residuals (80%) than there is in search residuals (65%).

### *Priming Correlations and Their Decay Over Time*

Sequential priming is an inevitable outcome of stimulus presentation and response. Implicit associations naturally occur when successive stimuli share common features (Maljkovic & Nakayama, 1994, 1996). These associations do not necessarily facilitate response, and negative priming will generally occur whenever stimulus attributes that were to-be-ignored become relevant. In addition, the motor aspects of the keypress response that is commonly used in this form of measurement will create correlations—fingers are activated and inhibited throughout a trial sequence. Priming correlations must be contributing to the spectral structure displayed in Figures 4 and 5, and consequently the role of priming is of considerable importance to this work. If priming were capable of producing spectra with increasing power at increasing trial scale, there would be little motivation to persist with this inquiry. There are two straightforward ways to evaluate the role of priming in the overall context of correlated fluctuations. The first is to determine experimentally how 1/f correlations vary under a set of treatments that are designed to suppress priming. The second is to explicitly calculate the range of influence that priming may have in situations where stimulus presentation is random. Both methods are presented here.

Two elementary discrimination experiments were conducted in order to obtain a corpus of priming correlations. Keypress response was purposely confounded with stimulus attribute in order that the correlations be maximal. The point here is not to disentangle the different ways priming influences data, but to separate priming from everything else. In the color experiment, a circle colored either red, green, or blue was presented on each trial. In the shape experiment, a black circle, square, or diamond appeared. In either case, the observer's task was to indicate with a keypress response which item appeared. There were seven conditions in each experiment defined by the size of a time delay that was interposed between trials. The time delays were 0 (self-paced trials), 0.5, 1.0, 1.5, 2.0, 2.5, and 5.0 s. The same eight observers completed two blocks of 540 trials in each condition in each experiment. The instructions were to respond as fast as possible but to try and keep errors down to about 10%. All of the observers were experienced psychophysical observers and were paid for their participation.

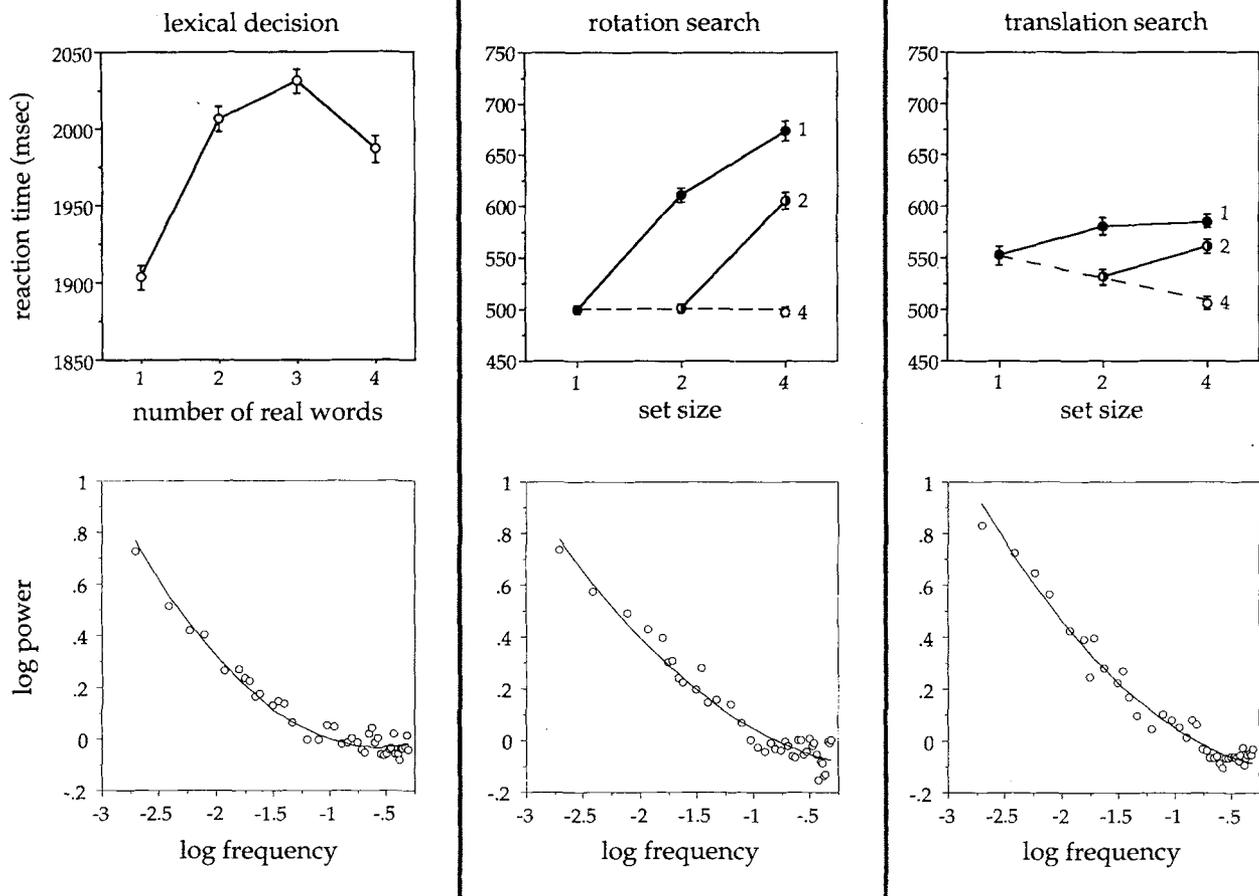


Figure 5. Plots of the means and residual power spectra for three additional studies of speeded response. In the search studies, the inset numbers and dot type indicate the number of targets present (filled = 1 target, half-filled = 2 targets, open = 4 targets). The dashed line shows trials on which all stimuli were targets.

Prior to analyzing the correlations between trials, all main effects (color or shape) were removed, the linear trends were removed, and each response sequence was recast in terms of  $z$  scores to equate the total variability across observers in all conditions. Priming effects were computed by sliding a moving window across the response sequences and computing the reaction time on trial  $N$ , given the stimulus presented (and consequently keys pressed) on trials  $N$ ,  $N - 1$ ,  $N - 2$ , and so on. A concrete example is clarifying here. Suppose that one is interested in determining the priming induced by two previous trials. In this experiment, there are three alternatives on each trial, so there are  $3 \times 3 \times 3 = 27$  possible stimulus histories that include a given trial and its two predecessors. Priming effects are computed by associating each reaction time latency with one of the 27 priming combinations. Across a sequence of 1,080 stimuli, each combination arises at random about 40 times, and the average latency for each combination is the desired statistic.

The main priming results for the color and shape experiments are displayed separately in Figure 6. The ordinate is the literal keypress sequence for the trial in question and for the two previous trials: R refers to the ring finger, M to the middle finger, and L to the index finger. The abscissa is the average reaction time  $z$  score

for the trial in question. Negative  $z$  scores correspond to response latencies that are shorter than average. A few examples elucidate the presentation of these data. First, consider the 0-s delay conditions where a new stimulus appeared immediately on a keypress response. Note that RRR, MMM, and LLL have the most negative reaction time  $z$  scores. These trials had a shape or color that also appeared on the two previous trials. This is the sort of priming effect that is expected—agreement between physical and response characteristics across trials leads to faster responding. The next most negative  $z$  scores correspond to trials of the form ABB, where the present trial is in agreement with the previous one but in disagreement with the trial two previous. The slowest trials are those having the form ABA, where there is agreement two back but disagreement one back. This pattern of data is an example of a pervasive attentional phenomenon referred to generally as inhibition of return. There are a number of reflection symmetries in Figure 6 that arise whenever priming magnitudes are invariant under substitutions such as  $L \rightarrow M$ ,  $M \rightarrow R$ ,  $R \rightarrow L$ . However, there is not perfect symmetry, as L and R are generally interchangeable but neither is generally interchangeable with M. For this reason, the response identities are displayed in favor of the more familiar stem and leaf plots (Luce, 1986).

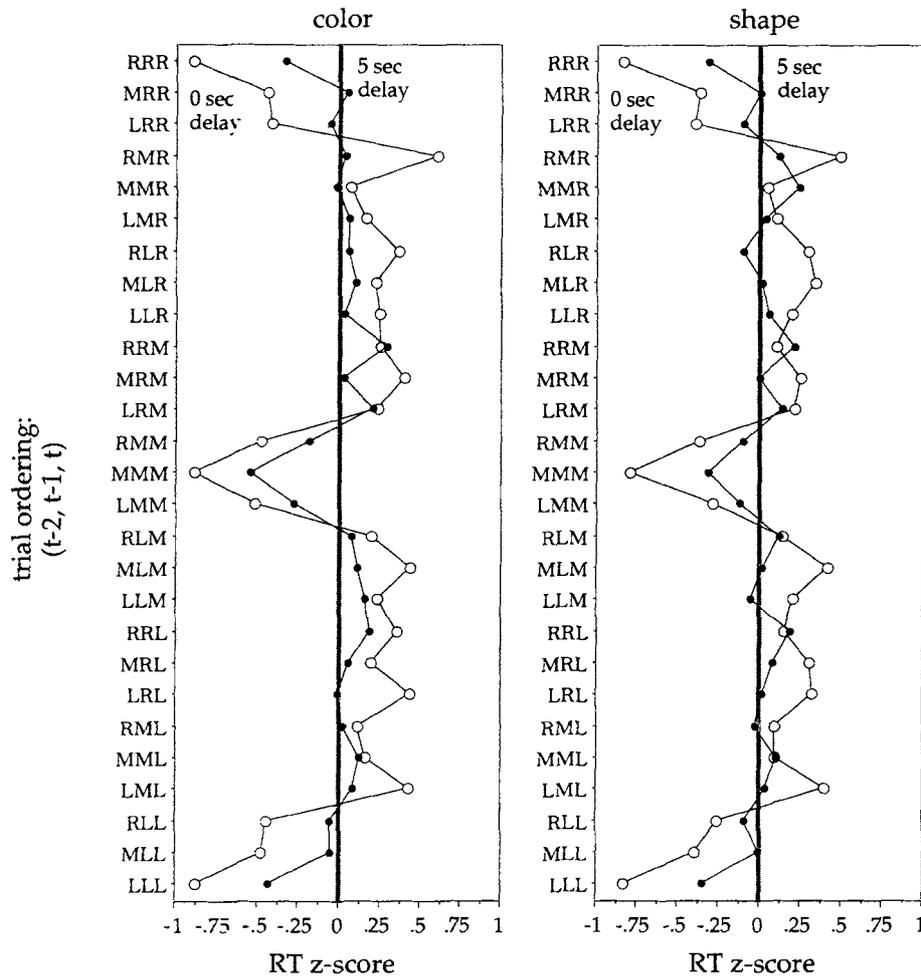


Figure 6. Second order priming in the shape and color discrimination studies. Average latency z scores are plotted as a function of the 27 different ways that three successive trials ( $t$  denoting the current trial,  $t - 1$  denoting the previous trial, and  $t - 2$  denoting two trials back) can be ordered in a three-alternative forced choice. L, M, and R indicate the correct keypress responses on the trials in question and refer to the index, middle, and ring fingers, respectively. Each panel shows priming functions for trials that were self-paced (open circles) and for trials that were delayed by 5 s (filled circles). RT = reaction time.

The central finding in this experiment is that sequential priming correlations decay with time. There is some amount of priming in the 5-s delay condition, but in both color and shape discriminations it is clearly weaker than that produced by self-paced trials. A measure of the total priming effect is a quantity referred to here as the *priming distance*. The priming distance is computed as the root mean square (RMS) z score and is formally the length of a data vector in the space of all three-tuples in keypress. Figure 7 displays the priming distance as a function of the interpolated time interval. For the most part, these functions are monotonically decreasing with delay time, implying the existence of lawful relation between time and priming magnitude. In fact, sequential priming decay is an example of cooling; the magnitude decreases exponentially over time.

In contrast to priming, the overall level of intertrial correlation does not decay with increasing intervals of time between trials. Figure 8 shows the power spectra of the exact same sequences that

were used to compute priming correlations. The power at all frequencies is intact in the delayed sequences. There is no evidence of whitening at low frequencies, which would be expected if the long-range correlations were also decaying. If there is any effect of time delay at all, it is to increase the amplitude of correlation at long trial intervals (low frequencies). This situation is true for both color and shape discriminations. This experiment shows, then, that priming can be dissociated from the long-term memory structures of interest here.

A more direct route to evaluating the contribution that priming makes to the overall level of correlation is simply to remove the priming correlations as completely as possible and then to compute the power spectra of the de-primed sequences. The number of trials (1,080) that were collected from each observer places a constraint on the depth to which sequential priming effects can be analyzed and therefore expunged. In a three-alternative-choice experiment, there are 81 possible stimulus combinations that arise in third-

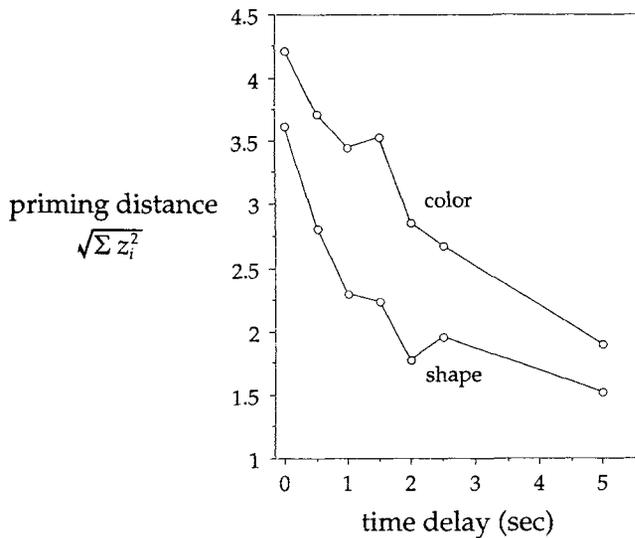


Figure 7. The decay of sequential priming as a function of imposed time delay between successive trials. The priming distance is the root-mean-square  $z$  score computed over the 27 combinations of three-tuples shown in Figure 6.

order priming where a trial and its three immediate predecessors are taken into account. On average, each combination occurs about 13 times in a sequence of 1,080 trials. In fourth-order priming, the number of possible combinations (243) is too numerous, and their individual occurrence too infrequent, to calculate reliable priming statistics. From a practical standpoint, then, priming effects can only be eliminated up to third order. However, this depth is sufficient because priming effects decay rapidly with trial separation. For example, although observers are faster when two or three stimuli are repeated, there is little benefit from a fourth or fifth repetition in reaction time. Exact calculations in both the color and shape experiments showed that 80% of priming effects are realized by consideration of only the two previous trials.

With these caveats, the removal of sequential priming from the color and shape experiments was straightforward.

Purging of priming correlations was done on each observer's data individually. The average latencies associated with the 81 stimulus combinations in third order were initially calculated. These average values were then subtracted on a trial-by-trial basis from the sequence of latency residuals. The procedure is formally identical to removing means, say, in mental rotation, except that now there are 81 conditions instead of 10. Consider, for example, a particular stimulus sequence RMRL that has a mean latency of  $X$ .  $X$  is subtracted from all trials on which L is the present response and where it was preceded by RMR—in exactly that order. This procedure effectively removed all priming correlations in all orders. The third order has been removed exactly, orders less than three are removed by virtue of being resolved by the third order, and higher orders are almost completely nullified because most of the priming in fourth and higher orders is due to lower order priming. The average power spectra of self-paced latencies with intact priming and with priming removed are shown respectively in the first and second columns of Figure 9. The top row refers to color discrimination and the bottom row to shape discrimination. The self-paced condition had the greatest level of priming and so is the relevant condition for this test. It is clear that extracting all of the priming correlations has little effect on the global structure of correlation. The power spectra with or without priming show the same increase with decreasing frequency, independent of the discrimination task.

The converse issue, of how much priming does contribute to latency spectra, may be addressed using Monte Carlo simulation. In this technique, the mean and standard deviation for each of the 81 stimulus combinations in the third order were calculated from a given observer's data, and then these distribution statistics were used to create a matched observer—one that makes only sequential priming correlations. The output of a matched observer was constructed by stepping through the stimulus sequence that was actually used in the experiment, looking at each trial and its three predecessors, finding the appropriate mean and variance for that combination, and then selecting a random number from a

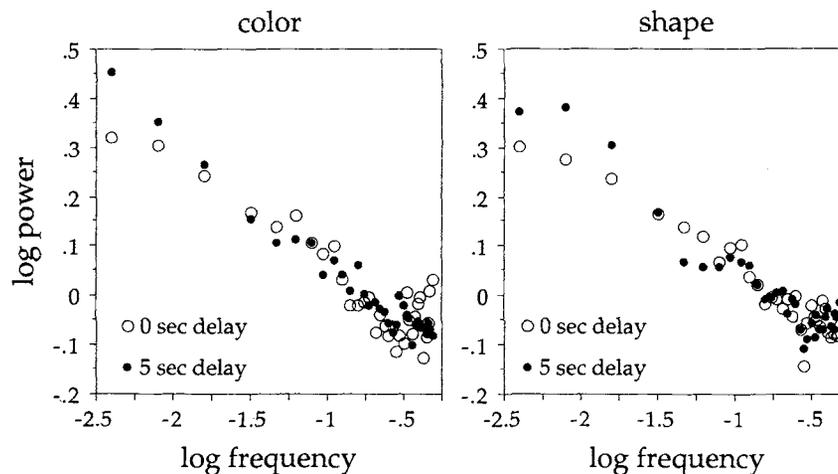


Figure 8. Power spectra of the residual fluctuations collected in the color and shape discrimination studies. Spectra are shown for both self-paced trials and trials delayed by 5 s.

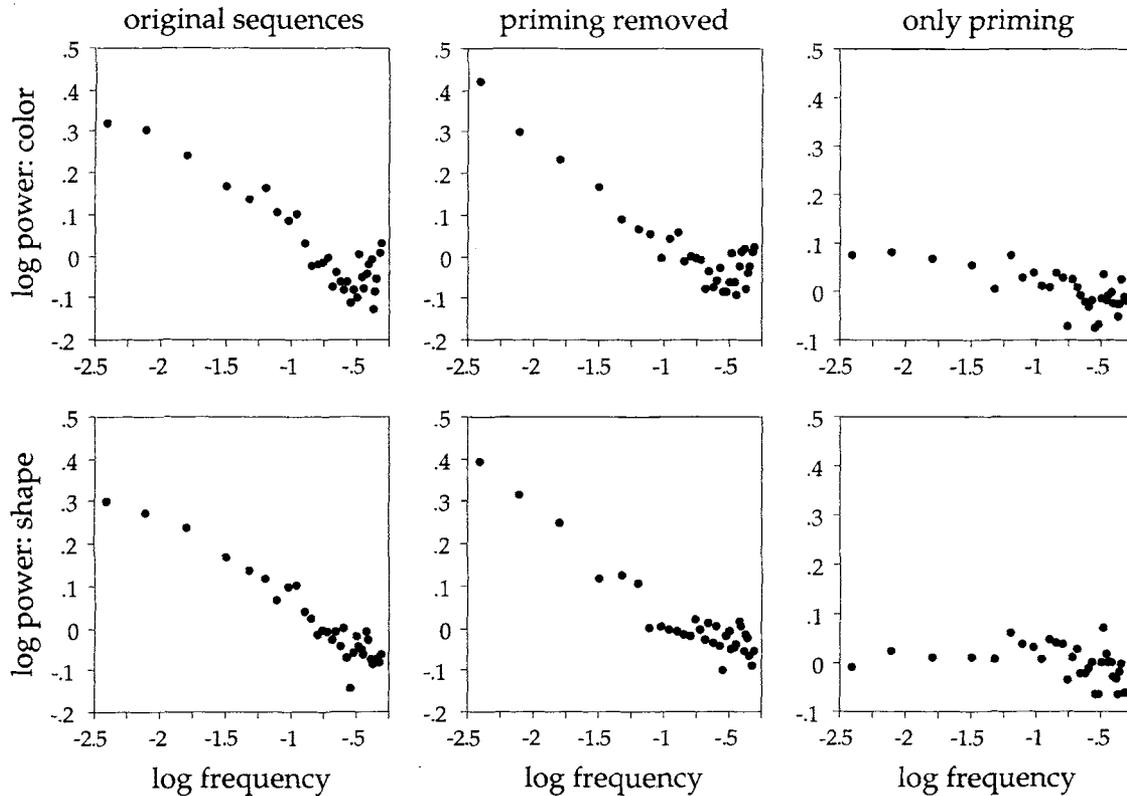


Figure 9. Influence of priming up to the third order on the power spectrum in the color (top panels) and shape (bottom panels) discrimination experiments. The first column shows the power spectra of the residuals in the condition of self-paced trials. The second column shows the power spectrum for residuals where the priming correlations have been removed to third order. The third column shows the power spectrum that would result were only priming correlations present.

normal distribution so defined (using other distributions with positive skew does not have an effect on the conclusions here). Matched sequences have exactly the same priming correlations as the original data to three orders, but that is all—they have no other source of correlation. The average power spectra for such matched observers are shown in the third column of Figure 9. The important feature in this representation is that the power spectra are flat at almost all frequencies except the very highest (small trial scale) where priming is active. Flat spectra are white noises, noises that have uncorrelated increments. This result implies that priming cannot produce correlations on long trial scales when stimulus presentation is randomized across trials, as was done in these experiments. If the stimuli are not presented at random, however, then virtually any structure is possible in the spectrum. Long time-scale hysteresis can arise, for example, even in systems where only a single previous state is encoded.

These experiments make the case that  $1/f$ -type correlations in response latency are not reducible to sequential priming. Priming effects have a very limited lifetime, decaying over a timescale of a few seconds. This lifetime sets an upper bound on the range of trials that can be correlated through priming. It also establishes a characteristic interstimulus interval beyond which priming effects are extinguished. The memory processes responsible for  $1/f$  noises, on the other hand, do not decay on the time scale of seconds.

Fractal correlations extend over scores of trials, and the imposition of delay times has little effect on spectral shape or amplitude.  $1/f$  noise is an example of a long memory process, whereas priming is inherently short range.

#### *Consistency of Mental Set in the Formation of Correlations*

There is something in the construction of the experiments so far described that leads to the emission of  $1/f$  noise. These experiments had few shared features and differed along every dimension that would be relevant to cognitive theory: stimuli, response, and task. At the most trivial level of analysis, the experiments all involved visual input, keypress output, and the demand that some sort of speeded decision be made. As is shown below, these properties have little to do with the formation of  $1/f$  noises, their presence being neither necessary nor sufficient for its production. The experiments also share one nontrivial feature: In any given study, the observers had only one task to perform. This constancy has psychological import in that it creates a consistent set of expectations and goals in the mind of the observer. That is, the observers are induced to adopt a particular mental set while serving in a block of trials. Recognizing that there is no complete description of what a mental set is within psychological theory, it

is patently a meaningful construct and involves the formation of particular perceptual organizations, specific styles of attending, and specific task-relevant representations. So although each experiment so far discussed induces a different mental set, there is continuity in that mental set over trials within any particular experiment. It is this continuity that I wish to inspect as potentially causal in the formation of  $1/f$  noises in response.

Mental set is manifestly an aspect of cognition that is not always constant. It changes whenever the goals and intentions of the observer change. There may, however, be another sense in which mental set is inconstant. In order to develop this second sense, one key idea that is fundamental to the theory of dynamic systems is required: Even when all of the parameters describing a system are held fixed, it may still exhibit unpredictable and complex behavior. Dissipative systems—those that are not energy conserving, and this includes all of biology—are capable of displaying point attractors, limit cycles, strange attractors, as well as fractional Brownian motions. The fluctuations that issue from such a system are intrinsic because they arise from its internal logic—the way it is put together. A system that is capable of generating intrinsic fluctuations does not need an external source of variability to exhibit variability in behavior. External noise, rather, typically has a disruptive effect by perturbing the underlying orbits away from their attracting states. If the constituent processes of mental set are not static fixtures, but have intrinsic fluctuations, then a tenable thesis is that these fluctuations are mirrored by latencies and observed as  $1/f$  noises.

The thesis that latency fluctuations provide a window into the dynamics of mental set is not easily proven. However, the thesis has sufficient definition that it makes a number of nontrivial predictions. The first prediction is that decorrelation of mental set will induce trial independence in response. Were the task parameters to change unpredictably so that the observer's representations and response mappings are forced to be uncorrelated, then any aspect of performance that relies on intrinsic fluctuations should be destroyed. Random assignment of task is formally equivalent to a large amplitude noise source, and if anything in this dynamical picture is correct, reaction times must decohere and lose their long-term correlations when the task is not constant across trials. If decorrelation of mental set does not whiten reaction time, then this framework is provably wrong. The second prediction is that  $1/f$  noise should be generic to response. It should be generic because it is a property of the thing being measured, not of the measuring tool. This issue will be dealt with extensively in the analysis of production and discrimination data.

The purpose of the following experiment was to create a context in which the observer would be unable to maintain a set of consistent representations that would suffice for all required responses. To be specific, this experiment created uncertainty by forcing the observer to discover what task he or she was in on a given trial. The experiment was done in two variations, each of which consisted of three parts: a mixed condition in which two tasks were switched at random and two control fixed conditions in which the task was constant. The mixed condition was designed around the notion of contingency; If X, then task Y is relevant, else task Z, where X, Y, and Z are the stimulus dimensions color, position, and shape. Contingency is an ideal construct for the creation of uncertainty; until a decision is made on X, the observer cannot create a representation of what is relevant in the stimulus.

The two variations of this experiment differed in the assignment of response dimensions (Y and Z) and cueing dimension (X). In the first variation, position served as the cue. Two outline boxes appeared side by side and so defined a local determination of left and right. Stimuli were the conjunction of a color (red, blue, green) and shape (circle, square, diamond). The observer's task was to make a speeded judgment of color if the object appeared in the left box or of shape if it appeared in the right box. In a block of 540 trials, the 18 possible combinations ( $3 \text{ colors} \times 3 \text{ shapes} \times 2 \text{ positions}$ ) appeared 30 times at random. The response keys were purposely mapped in a 2-to-1 fashion to prevent the formation of consistent response mappings. Circle and red required a keypress of 1, square and blue mapped to 2, and diamond and green mapped to 3. This task is naturally quite confusing because both color and shape are obvious to the observer but only one is relevant and this decision is based on a completely arbitrary (but fixed) positional assignment. This confusion is both the signature and the unavoidable consequence of not being able to maintain a consistent mental set. In the fixed conditions, the exact same stimuli were presented again in new random orders, except that now the observer had only to respond to variation on a single dimension and the response mappings were 1-to-1. So in this first variation, observers did two additional blocks of color discrimination ignoring position and shape and two additional blocks of shape discrimination ignoring position and color.

In the second version of this experiment, position and color reversed roles. The position dimension was augmented to three levels, again defined by adjacent outline boxes, so that it now included left, middle, and right. The color dimension was reduced to red and blue. The observer's task in the mixed condition was now to report on position if the object was red or on its shape if the object was blue. Again the response keys were confounded so that left and circle mapped to 1, middle and square mapped to 2, and right and diamond mapped to 3. In a block of 540 trials, the 18 possible combinations ( $3 \text{ positions} \times 3 \text{ shapes} \times 2 \text{ colors}$ ) appeared 30 times at random. Fixed conditions for this variation were defined as above. The exact same stimuli were presented in new random orders, and each observer completed two blocks of position discrimination ignoring color and shape and two blocks of shape discrimination ignoring color and position. In both experiments, six observers completed two blocks of self-paced trials with no feedback. The same observers participated in all three conditions of both variations. The order (fixed dimension 1, fixed dimension 2, or mixed 1 and 2) was counterbalanced across participants. In both variations of the mixed conditions, observers were given 100 practice trials so that they could learn the contingencies and response mappings. The mixed conditions were not cognitively simple tasks.

The data from these experiments were analyzed using the same tools and protocols that have been described earlier. In all conditions, cell means for the 18 distinct stimuli were removed. This subtraction nulls all possible main effects and interactions attributable to stimulus or response identity. Each block of trials was then linearly detrended, and each sequence of response latencies was then standardized so that it had mean zero and unit variance. The sequences were then ready to be analyzed for priming contingencies and spectral trends.

The priming contingencies displayed in Figure 10 illustrate the psychological costs of task and response switching. In this figure,

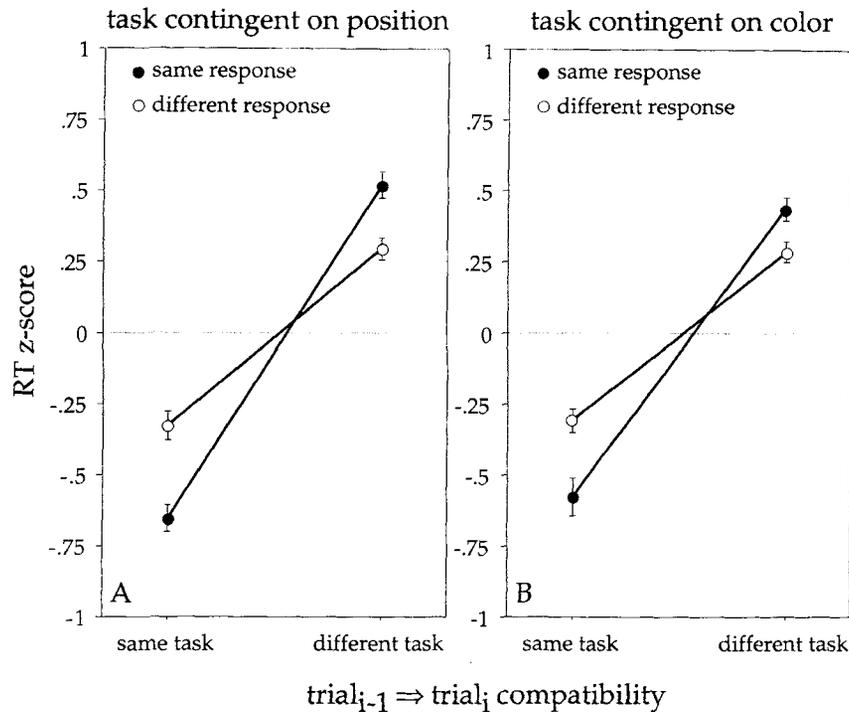


Figure 10. First-order priming patterns in studies where the task to be executed changed randomly from trial to trial. Panel A illustrates the case where position was the dimension of task conditionalization and where speeded responses were made on the basis of color or shape. Panel B illustrates the case where color was the dimension of task conditionalization and where shape or position formed the dimension of response. RT = reaction time.

the focus is only on the first-order contingencies and the data have been organized into a  $2 \times 2$  matrix defined by crossing task compatibility with response compatibility. The cells of this matrix are the average reaction time  $z$  scores on the  $i^{\text{th}}$  trial, contingent on whether the task defining the discrimination dimension was same or different and keypress response was same or different on the  $(i - 1)^{\text{th}}$  trial. Error bars depict between-subject standard error.

The principal effect in this experiment is the enormous influence that task compatibility has on response latency,  $F(1, 10) = 311$ ,  $p < .0001$ . Almost a full standard deviation separates response latencies when the task switched on two successive trials versus trials when the task remained the same. This result in itself demonstrates the difficulty observers have in, say, making a color discrimination on one trial and shape discrimination on the next. When tasks switch, attention must be deployed onto a new feature dimension and the meanings of the keypress responses change. This difficulty is not limited to a particular set of tasks; switching between shape and color (contingent on position) is just as difficult as switching between shape and position (contingent on color). This finding is not unexpected and really serves only as a check that the experiment succeeded in creating the desired incoherence in mental set. In addition to this main effect, the crossed interactions,  $F(1, 10) = 65$ ,  $p < .0001$  in Figure 10 reveal how response preparation is nested within task congruency. People behave as if they expect consistency in the implied dimension of change per se. If there is a change of task, responding is faster if there is also a

change in the required keypress. Responding is fastest if neither changes. The availability of a particular response is conditioned by consistency of mental set, not by consistency of response. This finding underscores the decoherence that is produced by task switching. Response consistency generally induces large facilitating priming effects. Here response consistency impairs performance whenever the task changes.

Whereas Figure 10 demonstrates the presence of strong sequential effects in mixed tasks, Figure 11 shows that task switching destroys the long-range correlations in the histories of response latency. Spectra are illustrated for fixed tasks done in isolation (Panels A1, B1, A2, B2) and when they are mixed together (Panels A3, B3). The two rows refer to the two variations in which the experiment was run. Figure 11 makes a simple but crucial point: Only when mental set can be consistently maintained are there long-term memory effects over the history of reaction time residuals. The inset numbers in each panel give the proportion of variance accounted for by a linear trend in the low frequency portion of the data (high frequencies are invariably whitened by motor fluctuations and are not shown here). The existence of long-term memory correlations is signified by a negative linear trend. This trend is evident in every case when the tasks are isolated so that mental set can be consistently maintained and is greatly reduced or absent when the tasks are mixed and mental set is forced to be incoherent. The flat spectra in Panels A3 and B3 suggest that discrimination and choice are not themselves suffi-

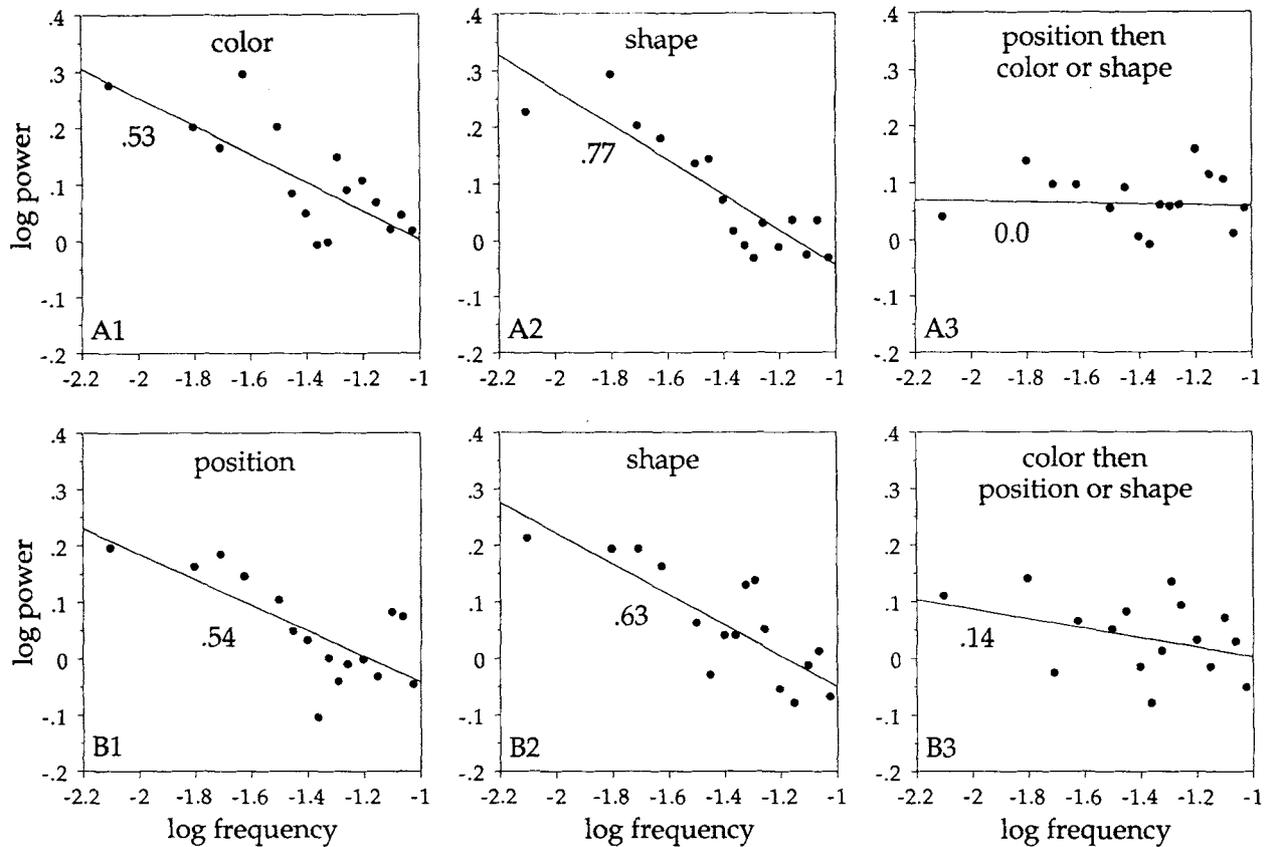


Figure 11. Power spectra of latency residuals at low frequencies for the two task-switching studies. The top panels show color discriminations alone (A1); shape discriminations alone (A2); and the mixed condition (A3), where color and shape discriminations alternated randomly conditional on position. The bottom panels show position discriminations alone (B1); shape discriminations alone (B2); and the mixed condition (B3), where position and shape discriminations alternated randomly conditional on color. The lines in each panel represent best fit linear trends, and the inset numbers show the percentage of variance accounted for by the trend.

cient for the formation of  $1/f$  noises; they must also be embedded in a consistent set of expectations and representations.

There is an alternative interpretation of spectral whitening under task mixing that deserves consideration. It may be the case that changing mental set does not interrupt anything like an intrinsic dynamic, and the observed decoherence in reaction time is simply due to the fact that there are two coincident patterns of correlated fluctuation that are resident in the individual histories of the separate tasks. Because the separate tasks are randomly interleaved, the two signals are mixed and so produce an uncorrelated white noise. However, were they to be disentangled, two separate correlated structures might be found, one for each of the tasks. It is a straightforward procedure to extract the reaction time history for each discrimination task from the composite mixture and to compute the power spectra for each extraction separately. The results of this analysis for both conditional variations are illustrated in Figure 12. The top panels show spectra for the two extractions from the color–shape mixture, and the bottom panels show the corresponding spectra from the position–shape mixture. In no case does the power increase at low frequencies; all four panels depict examples of white noise spectra. The implication is

that task mixing does not produce two separate correlated reaction time histories. Rather, the random interleaving of two tasks disrupts the memory processes that produce fractal  $1/f$ -type noises.

The picture that emerges from this experiment is distinctly physical. Mental set behaves qualitatively like any system described by equations whose solutions are functions of a set of control parameters. When mental set can be consistently maintained, the parameters are fixed at constant values and the solutions reflect the intrinsic dynamics. This is the default case in most experiments; the observers' expectations, focus of attention, object representations, and response mappings are fixed from the first trial. Task switching and the resultant loss of consistency in mental set is formally equivalent to intermittent and nonadiabatic (sudden) resetting of the control parameters. Under these circumstances, no system would behave coherently, and the dynamics in this case simply reflect the uncorrelated transients. A system as simple as a string pendulum will show this kind of behavior. It is not necessary to consider arcane examples of chaotic dynamics. In the sections that follow, this perspective is taken seriously and its consequences are explored in some detail.

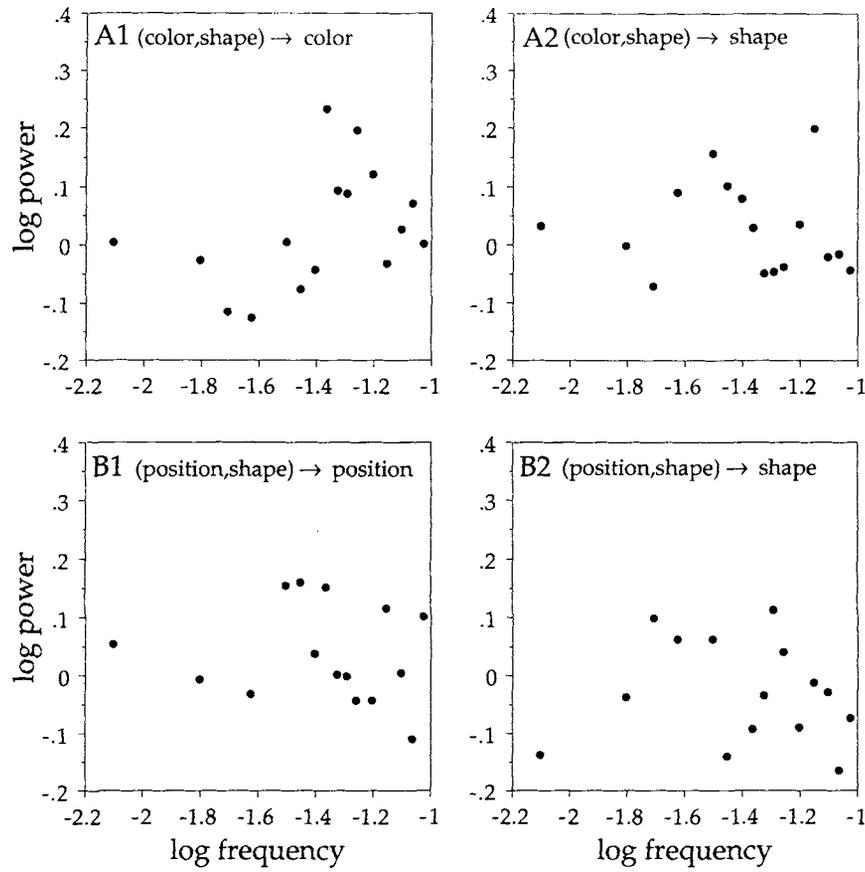


Figure 12. Power spectra of latency residuals at low frequencies for subsequences defined by discrimination type in the mixed task conditions. Panels A1 and A2 refer to the condition of color–shape discriminating mixing, and Panels B1 and B2 refer to the condition of position–shape discriminating mixing. Panel A1 shows the spectrum of the residuals on color trials, A2 shows shape trials, B1 shows position trials, and B2 shows shape trials.

### Fluctuations in Representation

Mental set is a highly complex construct and is not sufficiently well defined to provide a clear theoretical underpinning for understanding the formation of 1/f-type fluctuations. Mental set is, in a sense, too globally involved in decisions to be analytically useful. It influences the perceptual organization of the stimulus, it selects representations, it frames the context for decision, and it mediates the availability of response. There are patently too many processes involving mental set in typical experimental settings for individual sources of fluctuation to be isolated. The desire to reduce the complexity of experimental design leads to the following question: What is the simplest experiment that can be run on a behaving intact person? If residual structure is all that is required, then there is no reason to create different treatment cells in the first place. At most, one stimulus and one kind of response are necessary. Such a format is plainly unsatisfactory for use in discrimination paradigms because the uniqueness of the stimulus and response ensures that no discrimination need take place. However, stimulus–response designs are in no way mandated, and all of the prob-

lems of response uniqueness may be bypassed by eliminating the responding-to aspect of experimental designs and by just having people make things out of their imaginations. As a consequence, all of the experiments described below have the following design: The observer was instructed to produce some fixed quantity repeatedly, say 500 to 1,000 times. If the quantity was not immediately familiar, then an example was given prior to the observer’s efforts. There was no feedback, and the data were simply the history of what the participants took to be the quantity they were attempting to replicate. In this way, the moment-to-moment fluctuations in their representations were produced with a minimum of interference. There is, however, always some interference because the intention to perform an act is not identical to the act itself. In all of the experiments described here, some hand movement was required, and there is inevitably some error associated with motor performance that is independent of what the movement signifies. This methodology provides the purest titration of fluctuations in representation that can be behaviorally acquired. Consistency of mental set is reduced to the maintenance of a single intention.

### Fluctuations in Representations of Time

In Gilden et al. (1995), observers were asked to make a keypress every time they thought a target interval of time had elapsed.<sup>3</sup> In this set of experiments, the same six observers made repeated estimates of intervals having the following durations in seconds: 10.0, 5.0, 1.5, 1.0, 0.5 and 0.3. The number of estimates was 1,000 in all cases except for the 10-s condition, where mercy required that the number be limited to 400. Each observer contributed one sequence in each target condition following a 1-min presentation of the target interval from a metronome. The metronome was not on during the collection of data. The sequences were timed so that the keypress that signaled the end of one interval also initiated the timer for the next. In this way, the observer could tap his or her finger to a rhythm for those targets that permitted such an organization.

The average power spectra for all conditions are shown collectively in Panel A of Figure 13. The spectra are labeled by the target duration. In this figure, the overall scaling of power is arbitrary, but the same scale is used for all data. Timing errors are roughly Weberian (error proportional to target magnitude); consequently, there is more spectral power in production errors at longer durations. The frequency scale has been placed on a hertz (inverse seconds) scale by normalizing the inverse trial frequency by the target duration. This is the only experiment where it makes sense

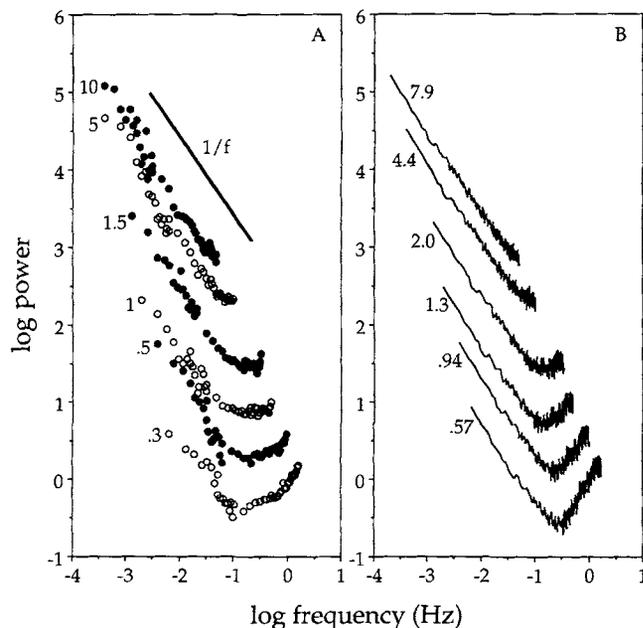


Figure 13. Power spectra of temporal duration estimates and a model. Panel A displays the average spectra from six experiments as a function of temporal frequency (in Hz) and the magnitude of the target interval (in seconds). Also shown is a line depicting an exact  $1/f$  power spectrum. Filled and open circles are used alternately with target interval so that the spectra are adequately individuated. Panel B shows the best fits to the data from a two-source model on the basis of separate cognitive and motor components. The different curves are indexed by the ratio  $\sigma$  (cognitive noise)/ $\sigma$  (motor noise) that provided the best fit to the data in the least squares sense.

to use hertz as a frequency scale because in this single case the data are estimates of temporal intervals.

Estimation of temporal duration proved to be an efficient  $1/f$  noise source. In every target condition,  $1/f$  noise dominates the low frequencies. Descending from top to bottom, at frequencies less than 0.1 Hz, best fit lines to the power spectra had slopes  $-1.1$ ,  $-1.0$ ,  $-1.1$ ,  $-0.90$ ,  $-1.2$ , and  $-0.94$  respectively. There is, however, an equally clear non- $1/f$  component of noise that is manifest primarily at high frequencies. The non- $1/f$  component is increasingly dominant at shorter target durations where responding is naturally at higher frequency. In order to make sense of these data, a variation of the two-source model was used. This model is formally identical to that used in standard treatments of timing error (Wing, 1980; Wing & Kristofferson, 1973). In a simple model of timing, the production of temporal intervals is composed of two parts: an internal clock (C) that mediates the judgment of time passage and a motor program that actuates the responses that signal the beginning and ending of each interval. The motor program in this model does not operate instantaneously, and all responses have an assigned motor delay (MD). In terms of these two components, the  $j^{\text{th}}$  observed interval  $I_j$  is written as

$$I_j = C_j + MD_j - MD_{j-1}. \quad (3)$$

The difference in motor delays arises from the particular boundary condition typically used in timing studies; the ending of the  $(j-1)^{\text{st}}$  interval is also the beginning of the  $j^{\text{th}}$  interval. Recall that in the analysis of reaction time, the motor component was contained in a single term because the timer began not with a keypress but with the presentation of a visual stimulus.

This model is algorithmically completed by specifying the functional forms of the clock (C) and motor delay (MD) components. In the analysis of reaction time, a rather general model was used that was composed of a source of correlated fluctuation and a source of white fluctuation. The spectral power of the correlated source was not specified and was left as a free parameter. The spectra in this experiment are virtually uncontaminated by white noise at low frequencies, and it is possible in this case to fix the power to be exactly  $-1$ : That is, the spectral power of fluctuations issuing from the internal clock (C) is exactly proportion to  $1/f$ . As a consequence, any white noise that enters into the estimates does so through motor error. This model is highly constrained and has but one free parameter: the ratio of the standard deviations of the fluctuations emitted by the two sources. Simulations of the optimal models are shown in Panel B of Figure 13. The inset numbers give the derived ratio of standard deviations that provided the best fit to the data in the least squares sense.

It is evident from Figure 13 that this simple timing model is quite successful at describing the data; the model spectra look like the data. The model also makes it clear why the spectra take on the particular shapes they do at the various target durations. At long target durations, the magnitude of the clock error is larger than that of the motor error, entailing that the spectra are dominated by  $1/f$

<sup>3</sup> The observers were free to count because there was no way to prevent them from doing so in the absence of some secondary attention-demanding task. The presence of a secondary task was not appropriate here, and so counting was tolerated although it was not encouraged.

noise. At shorter durations, this inequality changes direction and the difference signal of white noise predominates. A difference signal does not have a flat spectrum but one that increases (linearly) with frequency (the Fourier transform of the derivative of any function is the Fourier transform of that function multiplied by the frequency). This fact accounts for the increasing power at high frequencies, where the motor terms make substantial contributions to the spectra.

The derived ratios of standard deviations that result from fitting this model to the data have a straightforward psychophysical interpretation. If the motor error has a constant variance independent of the target duration, then the derived ratios are, up to a constant of proportionality, the standard deviation of the internal clock source. The motor error arises from the point of decision to the moment of keypress, and this interval should depend weakly, if at all, on the absolute magnitude of the duration interval. Under this assumption, the implied clock source is almost exactly Weberian: The target durations are linearly related to the derived standard deviations ( $r^2 = 1$ ). There is some controversy over whether timing errors are Weberian (Allan, 1979; Wearden, 1991), but this analysis provides compelling evidence that at least the internal clock is Weberian.

The clarity of these spectra, were they presented in isolation, might suggest that 1/f noise is in some way related to internal clocks. However, similar correlated fluctuations have been seen in reaction time, and even though time is also the mediating variable in measures of speeded judgment, reaction time is not a measure of perceived duration. These data, rather, acquire their shape because of intrinsic fluctuations in representation, and it matters little what is being represented. Further evidence for this claim is presented in the next section, where representations of distance, angle, and force are analyzed.

### *Fluctuations in Representations of Space and Force*

Production data should generally be rife with 1/f noise if it is true that the production paradigm offers the most direct access to the dynamics of representation and that this dynamic has a 1/f noise signature. Observing these noises should simply be a matter of finding appropriate measurement devices. Digital tablets, rheostats, and pressure plates work admirably in this regard. These devices were used respectively for estimates of spatial interval, angle of rotation, and applied force. Mark Schmuckler of the University of Toronto collected the rotation and force data. In the following designs, the trials were self-paced and no feedback was given.

*Spatial intervals.* Six participants estimated distances of  $\frac{1}{8}$ ", 1", and 4" 1,000 times each in three separate sessions. Estimates were made on a digital tablet that had a resolution of 1,000 intervals per inch. Each trial began with a pen placement on a premarked position, followed by a second pen placement at what the observer took to be the target distance.

*Rotation.* Participants estimated rotations of 45°, 90°, and 180° using a hand-held rheostat with a resolution of 2,048 intervals in 360°. The rheostat was placed behind a partition so that no visual feedback was available and was returned to the vertical position prior to each trial. The vertical position was well defined, and the angular interval was marked off in the clockwise sense by

the participants, who were all right-handed. In the 45° condition, nine participants completed between 1,700 and 3,000 trials. In the 90° condition, nine participants completed between 1,600 and 3,200 trials. In the 180° condition, six participants completed between 1,400 and 2,300 trials.

*Force.* Participants were instructed to press down on a force plate at a comfortable but firm level of force. The participants understood that their selected level of force should be chosen so that it could be repeated without fatigue. Six participants completed between 1,200 and 2,000 trials.

The power spectra of the production sequences were computed as before following linear detrending. The average spectra are shown in Figure 14 together with lines that have a slope of  $-1$ . The spectra in each panel are quite similar, and they are all good examples of 1/f noise over a wide range of frequency. Unlike the fluctuations of reaction time latencies, there is very little whitening in these noises. There was also negligible whitening in the estimates of temporal intervals that could not be attributed to motor errors. These spectra provide confirming evidence that representation has intrinsic fluctuations, and that these fluctuations have behavioral consequences. An experimental setting was provided where the processes of representation had the most favorable opportunity to be observed, and it is in this setting that the clearest examples of 1/f noise are recorded.

The high-frequency portion of the spectra in Figure 14 is also of interest. In each condition, there is a critical frequency above which the spectrum whitens. The same phenomenon was observed in the timing studies except there the keypress boundary condition (the ending of one trial starts the timer for the next) led to upturns in the spectrum. The scaling of this transition frequency with the magnitude of the target quantity is not the same in angle and distance estimation. The transition frequency for spatial intervals decreases as the interval decreases. This behavior was also observed in the estimates of temporal intervals and has the same interpretation: The cognitive component of the error is roughly Weberian, and the motor error, in this case attributable to hand positioning, is roughly constant in amplitude. Constancy of the motor error makes sense in the case of distance estimation in that the pointing errors associated with pen placement on a digital tablet should be independent of the absolute position of the pen—so long as the arm or hand is not strained into an unnatural position. In contrast, the transition frequency for estimates of angle is roughly the same at 45°, 90°, and 180°. Analysis of the rotation data revealed that the growth of error was roughly Weberian, with a coefficient of variation of about .12. The constancy of the transition frequency in the spectra implies that both the white and 1/f contributions are growing at this rate. The reason for this must ultimately have to do with the obvious fact that the hand is not in the same position when it is at rotated at different angles and is under greater stress at larger angles of rotation.

### *Fluctuations in the Accuracy of Discrimination*

One of the principal problems in using physical concepts to interpret psychological data is the absence of a uniform and coherent system of reference. Reference is a problem for all sciences, but in psychology it is especially severe. So for example, the claim that the river Nile exhibits 1/f height fluctuations is

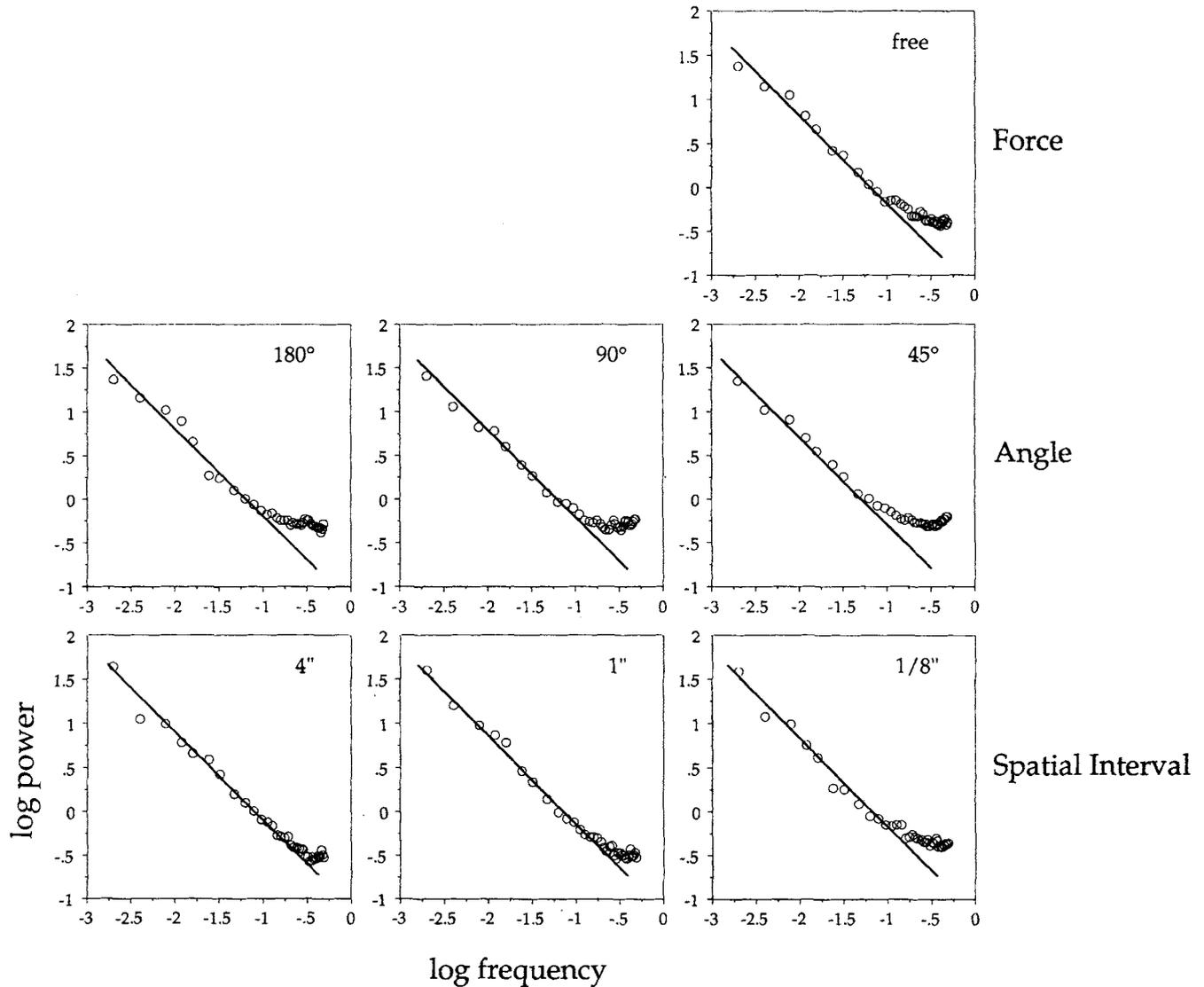


Figure 14. Power spectra for estimates of applied force, angular rotation, and spatial intervals. Each data set is accompanied by a line that depicts a pure  $1/f$  noise.

physically sensible because both the river Nile and its height have meaningful referents. The basis for such a statement would presumably be the history of fluctuations as measured by some instrument, but the reference is ultimately to a body of water and not to the history of a measuring device. In contrast, the assertion that reaction time or production exhibits  $1/f$  trial fluctuations does not succeed in making reference to the distal source whose behavior is being commented on. This issue is particularly important here because these data are interesting only if they are a signature of a cognitive process and not an artifact of methodology. Cognitive psychology is, however, completely informal, and there is no fundamental theory of discrimination or decision that can be written down in such a way that one may point to a term and identify it as the fluctuating quantity. In the absence of an explicit theoretical understanding of what constitutes a cognitive act, the

strongest implications about process arise from converging operations, or more formally, from demonstrations of measurement consistency.

Consistency between speed and accuracy measurement is entailed by any theory that associates  $1/f$  noises with the processes of representation. If there is in fact a dynamic intrinsic to information acquisition that *causes*  $1/f$  fluctuations in latency, then this dynamic should also cause similar fluctuations in accuracy. In both methods, the observer's task is essentially the same: to interpret the stimulus and to make a decision. The demand for consistency is a strong test of the theoretical framework presented here: that  $1/f$  fluctuations are the dynamic trace of representation. A negative finding would imply that the correlations that have been observed in reaction time and production are probably due to artifacts specific to these para-

digms.<sup>4</sup> The issue of whether accuracy fluctuations harbor 1/f noises is not a question that has been previously addressed, and there is nothing in the methodology of two alternative forced choice (2AFC) that decides this question.

### *Streaky Performance in Signal Detection*

The impression that positive correlation leaves on a trial sequence depends on what is being measured. Positive correlation in the measurement domain of response latency generates nested waves. These waves may be of sufficient amplitude that they are visible to the eye. Positive correlation in accuracy measurement, however, does not generate waves for the simple reason that accuracy is a discrete binary variable. Instead of waves, positive correlation in a binary variable will induce nontrivial conditionalization between outcomes. Deviating from the standard terminology and referring to correct responses as *hits* and incorrect responses as *misses*, positive correlation is signified by hits tending to follow hits and misses tending to follow misses. When the focus is on an entire sequence of discriminations in a repeated measures design, positive correlation is manifest as clustering of like outcomes. Clustering is what waves look like when discretized. In the deployment of motor skills, the phenomenon of clustering is referred to as *streakiness* (Gilovich, Vallone, & Tversky, 1985). A clustering process yields fewer runs of greater length than would be expected from a Bernoulli process operating at the same overall hit rate. The Bernoulli process is the discrete equivalent of the white noise generator. It is a process defined by stationarity (trial independence) in the instantaneous probability of a hit.

Gilden and Gray Wilson (1995) investigated the structure of outcome sequences in a variety of two alternative and two interval forced choice signal detection tasks, which are described briefly below. In each of these studies, between 4 and 10 participants completed on the order of 1,000 trials over two sessions. Every participant was individually calibrated prior to each testing session so as to maintain accuracy near 75%.

*Ovateness.* Random dot kinematograms depicted either spheres or egg shapes. The task was to categorize each stimulus as a sphere or an egg.

*Distance ratio.* Two line segments of different sizes were shown, each with a dividing line. One segment was a standard that served as a reference. The task was to decide whether the dividing line on the other segment should be moved to the left or right to achieve an equal ratio of internal segments.

*2Fractal.* Two line drawings of fractional Brownian motions were shown. The task was to decide which noise was smoother.

*1Fractal.* Line drawings of fractional Brownian motions were created from two classes. The task was to decide from which class the noise on a particular trial was selected.

*Tone 2IFC.* Two intervals of acoustic white noise were presented. In one interval, a pure tone was also embedded. The task was to locate the interval containing the tone.

*Tone 2ears.* Acoustic white noises were presented to both ears independently. In one ear, a pure tone was also presented. The task was to locate the ear containing the tone.

*Luminance.* Two adjacent squares differing in luminance were presented for 500 msec. The task was to judge which was brighter.

*Orientation.* Two lines, one vertical and one tilted to the right, were presented for 16 msec. The task was to decide which was tilted.

*Side missing.* An outline square that had either the left or right side missing was presented for 70 msec. The task was to identify the missing side.

*Flash.* Two squares were shown side by side. During a 16-msec interval, one square brightened. The task was to decide which square this was.

The outcome sequences of hits and misses were first analyzed in terms of the number of runs they contained. For a given hit rate (proportion of correct responses), the probability that a Bernoulli process would generate the observed number of runs or fewer was computed exactly (Hays, 1988) and then converted to a *z* score—the runs *z* score—by inverting the cumulative normal distribution. In this way, every outcome sequence is associated with a runs *z* score, and Figure 15 displays their distributions by task. Remember that positive correlations between outcomes (hits tending to follow hits) leads to fewer runs and so to negative run *z* scores. It is evident that positive correlation and streakiness are the rule in visual and auditory discrimination. Ovateness categorization was the single task that generated outcomes consistent with Bernoulli trials.

One of the central findings of this work was that the magnitude of the runs deficit was related to attentional demand. The bottom four tasks in Figure 15 are all of one kind. At superthreshold contrasts, these discriminations would have led to so-called pop-out in singleton search, and they generate a common level of streakiness at threshold contrasts with run *z* scores near  $-1$ . The experience of pop-out is formalized in singleton visual search by the independence of target acquisition times from the number of distractors. Independence of response time from set size implies that targets and distractors are analyzed in parallel with little (potentially zero) capacity limitation. This limit is referred to as *preattention*, and there is evidence that it exists at the threshold levels of contrast where discriminations in 2AFC are obtained (Palmer, Ames, & Lindsey, 1993). The other tasks all involve some type of shape, temporal, or direction discrimination and would show signs of capacity limitation.<sup>5</sup> The capacity-limited discriminations differ from the pop-out variety in two ways: They are not as streaky (*z* scores are closer to the expectation of trial independence) and they are more variable. The implication here is that capacity limitation either suppresses the memory processes that create positive correlation or that the effortful usage of attention acts as a source of white noise. Preattention is the lower bound

<sup>4</sup> For example, there may be instability in the setting of decision criteria. Speeded judgment requires that an information threshold be passed before judgment can be made, and there is no reason to believe that such thresholds are fixed.

<sup>5</sup> It must be recognized that the preattentive-focused attention distinction has an uncertain empirical foundation and that attentional capacity is probably graded in a continuous manner. Nevertheless, luminance and orientation differences clearly support highly efficient search, and this is not the case with subtle shape differences such as interval ratios or fractal roughness. Ultimately, it may be that analysis of outcome fluctuations provides a better characterization of attentional resource usage than search methods. At least run counting is not muddled by speed-accuracy trade-off, favored position effects, or uncertain classification of reaction time-set size functions.

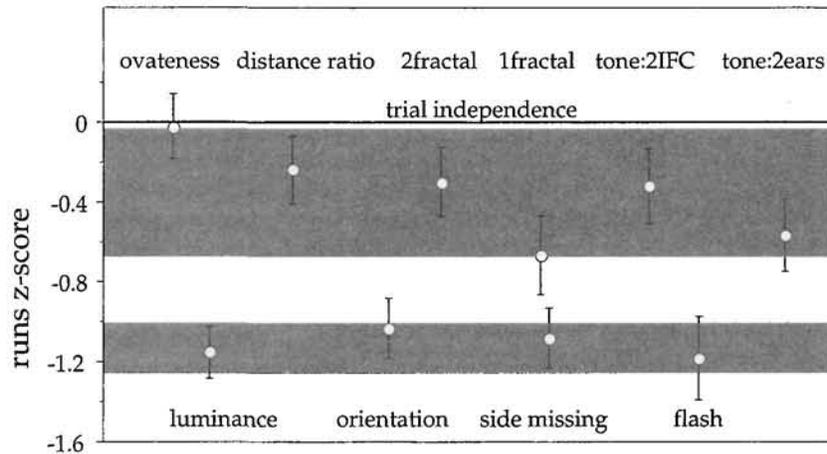


Figure 15. Plot of the runs  $z$  score for 10 signal detection studies. Six tasks requiring focused attention occupy the upper gray region. Four tasks that would lead to pop-out at superthreshold levels of contrast occupy the lower gray region. Negative  $z$  scores correspond to runs deficits relative to the expectation of trial independence.

to capacity limitation and so forms an equivalence class in the level of outcome clustering among all such discriminations.

These results make a clear and consistent case that memory processes create nonstationarity in perceptual sensitivity. The act of making a discrimination at one time influences the outcome probabilities at later times. Stimulus and motor priming are incapable of creating the observed correlations for the same reasons they could not in latency. Sequential priming is a local process that has minimal impact on scales larger than about five to eight trials. As is shown immediately below, memory processes are effective at creating nonstationarity on scales of at least 50–100 trials. Again, mechanisms that have inherent persistence are implicated. Potential mechanisms include fatigue, learning, and wavelike rhythms in attention. Gildden and Gray Wilson (1995) developed a series of statistical analyses that eliminated fatigue and learning from this set and presented evidence in favor of a wave model. Here the statistical analysis is substantially broadened, the wave model is shown to be inadequate, and the issue of nonstationarity in 2AFC outcome is raised in an entirely new context. Decisions about models require more complete descriptions of sequence structure than is given by the runs  $z$  score. The following section develops a variety of statistical tools for analyzing patterns of outcome in order to provide benchmarks that models have to meet.

### The Fine Structure of Signal Detection

There are a variety of ways of characterizing the statistical structure of binary outcome sequences. Gilovich et al. (1985) used four groups of statistics in their analysis of streaks in human performance; the number of runs, the serial correlation, the set of conditional probabilities, and the hit density. The runs  $z$  score is equivalent to the serial correlation at a lag of 1 (when transformed to the Fisher  $Z$ ), and although useful as a simple measure of streakiness, it does not contain much information about the way sequences are put together. Conditional probabilities (probability of a hit given a hit, given two hits, given a miss, and so forth) have

the power to completely characterize sequence structure, but they are most useful when the effects of nonstationarity are limited to a few trials as the number of conditionals increases geometrically with look-back time. Priming effects, for example, have short lifetimes and are best summarized in terms of conditional probabilities. This is not the case for discrimination outcome, and ultimately conditional probabilities offer more information than can be analytically assimilated. The hit density is a medium resolution statistic that provides excellent discrimination between theoretical models and is used extensively here. As this statistic is not in wide use, a brief description of its formulation is required.

The hit density measures the probability of encountering a subsequence of size  $N$  with  $K$  hits, where  $0 \leq K \leq N$ . Its dependence on scale ( $N$ ) makes it a much richer statistic than the runs count, and its pooling of hits makes it much more tractable than statistics that require keeping track of the exact order in which hits and misses occurred. Hit densities are computed by partitioning an outcome sequence defined by a block of trials into non-overlapping windows of size  $N$  and counting the number of such windows that contain 0, 1, 2, . . .  $N$  hits. Hence, there are  $N + 1$  statistics associated with each scale  $N$ . Hit densities can be computed exactly under the null hypothesis of independent Bernoulli trials, and this provides the appropriate comparison for assessing local effects of nonstationarity. It is important to note that here, as in the computation of the expected number of runs, each outcome sequence is compared with a Bernoulli process uniquely specified by that sequence's average hit rate. The departure of the observed hit densities from a matched Bernoulli process is referred to as the *density difference*.

The hit density is not the only measure that has scale sensitivity without the burden of exact order information. The power spectrum also has these properties, and it provides a more highly resolved description of sequence structure. Additional resolution comes at the price of greater variability, and this makes the statistic less suitable for detailed model fitting. The power spectrum is used here primarily as a check that there are no salient aspects of the data not captured by the best-fit models. That the outcome signal

is binary makes little practical difference in Fourier transformation. The power spectrum is still computable and meaningful, and it comes equipped with a unique realization for the null hypothesis of stationarity: the white noise spectrum with a slope of zero. Figure 16 displays both forms of medium resolution statistic across the four preattentive discrimination tasks (flash, orientation, side missing, and luminance). The density differences are displayed here at five window sizes,  $N = 5, 10, 15, 20,$  and  $25$ . Each window-size  $N$  is resolved by  $N + 1$  counts: the number of

windows of size  $N$  that contained  $K = 0$  hits, 1 hit,  $\dots$ ,  $N$  hits. The  $N + 1$  counts are displayed in order of increasing  $K$  as probabilities relative to the expectation of a Bernoulli process.

The observed density differences depart from the expectation of Bernoulli trials primarily through the formation of twin peaks surrounding a trough near  $K = .75N$ . This structure is caused by the way that positive correlation associates hits in sequences where the modal value of  $K$  is  $.75N$  (the average hit rate being  $.75$ ). A clustering process can be thought of as a type of diffusion where

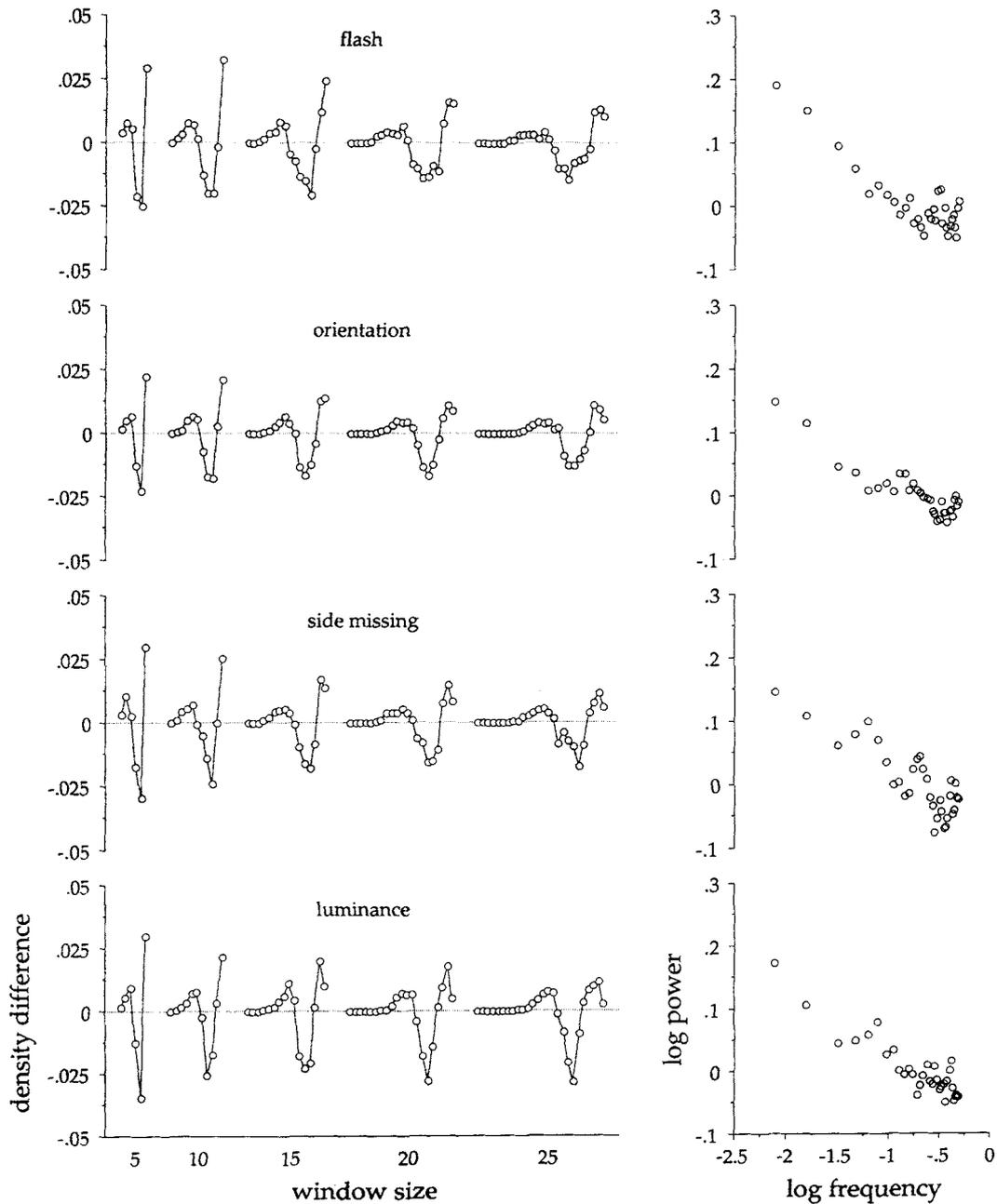


Figure 16. Density differences and power spectra for the four preattentive discrimination studies. The density differences are shown at five window sizes,  $N$ . There are  $N + 1$  points plotted at each window size—the relative probability of encountering a window of size  $N$  with  $K = 0, 1, \dots, N$  hits.

hits migrate toward each other. Diffusion causes some of the  $K = .75N$  windows that would have existed under trial independence to become enriched with hits so that they are transformed into windows with  $K > .75N$ . Because the number of hits in a sequence defined by a given base rate is fixed, every time a window with  $K > .75N$  is created from a potential  $K = .75N$  window, a window with  $K < .75N$  results. In this way, streakiness leads to peaks on either side of a dip at  $K = .75N$ . The curves are likewise constrained to approach zero at small values of  $K$  because hit-empty windows are neither expected nor observed when the basal probability of a hit is .75.

Figure 16 illustrates two key properties of preattentive discrimination outcome: the sequence structure is task independent and scale free. Both results suggest that outcome histories are governed by a lawful generating process. The logic of this process is not given by any psychological theory, but it may be deduced using Monte Carlo simulation. To be specific, the goal here is to specify a rule structure that generates outcome sequences that are statistically indistinguishable from the data actually received. Gilden and Gray Wilson (1995) developed a wave model of nonstationarity that was quite successful in creating simulated sequences that looked like preattentive 2AFC data on small trial scales. Wave models, however, create line spectra and for this reason must be discarded. A successful model of 2AFC outcome must be able to produce a continuum of spectral energy that does not select preferred scales of fluctuation.

### A Reaction Time Model of Discrimination

Latency residuals and discrimination outcomes are both examples of a noise process that is positively correlated and scale free. This congruence is presumably not coincidental but is merely the result of using multiple measurement tools to observe a single cognitive structure. That is, streaks may be what latencies look like when accuracy measures are used to probe cognitive activity. If this interpretation is true, then speed and accuracy measurements are consistent at the level of fluctuation. This kind of consistency is much stricter than implied by speed-accuracy trade-off and would provide necessary empirical support for the claim that  $1/f$  noise is an inherent aspect of decision and discrimination. Trade-off involves only a comparison of means, whereas the analysis of fluctuations involves the full correlation function at all lags. In formal terms demonstrating consistency comes down to finding a relation between latency and accuracy that maps  $1/f$  noises into streaks. The following Monte Carlo simulation provides a numerical recipe for constructing a relation that has exactly these properties.

The simulation takes as its point of departure the notion that both speed and accuracy reflect a common cognitive source and that it makes sense to identify fluctuations in speed with fluctuations in accuracy. Insofar as there is no empirical or theoretical work to confirm or deny this approach, the identification is made with the sole purpose of determining its consequences. The key step in the simulation is placing latency residuals on the interval  $[0, 1]$  so they can be interpreted as probabilities. This was done through the following linear transformation:<sup>6</sup>

$$p_i = p_0 + \Delta e_i, \quad (4)$$

where  $p_i$  is the probability that the  $i^{\text{th}}$  trial yields a correct discrimination and  $e_i$  is the  $i^{\text{th}}$  residual latency in a reaction time sequence. This mapping permits the construction of simulated binary outcome sequences that fluctuate like reaction times but are interpretable as the outcomes of 2AFC discrimination. The intercept,  $p_0$ , determines the average hit rate (proportion of successful trials) in a simulated outcome sequence, and  $\Delta$  determines the magnitude of its autocorrelation relative to that already present in the latency residuals. Furthermore,  $p_0$  is a property of individual sequences and may be chosen in any number of ways so long as there is variability in hit rate. It makes little difference if  $p_0$  is selected randomly on the interval  $[0.5, 1.0]$ , the range of hit rates in 2AFC, or if it is chosen so that the simulated sequences have the same hit rate distribution as the observed sequences. The latter choice was made in the simulations presented here. The scaling factor,  $\Delta$ , is the single free parameter in these simulations, and it controls the transfer of the  $1/f$  signal in latency into nonstationarity in discrimination.  $\Delta$  is not a property of individual sequences but is fixed as a constant over the entire ensemble. Furthermore, the only constraint imposed on  $\Delta$  is that both simulated and observed sequences have the same average runs  $z$  score.

There is a practical issue of how constraints on hit rate and run count affect the medium resolution statistics. That is, once it is demanded that simulated sequences have the same hit rate distribution and the same average runs  $z$  score as the data, how much freedom is there for the simulation to fail to produce the rest of the statistics as well? This question was addressed by simulating a test case where the constraints were maximized: Random sequences were constructed to match exactly both the runs  $z$  score and hit rate,  $p_0$ , of observed sequences. This constraint differs from that used in the actual modeling by requiring that the simulated sequences not only have the same average runs  $z$  score as the data, but also the exact runs  $z$  score distribution. From a formal standpoint, this simulation determines how much of the medium resolution statistics are captured by the lowest order moment (average hit rate) and lowest order correlation (the serial correlation at a lag of 1).

The ensemble of matched sequences was created using a brute force algorithm. The procedure consisted of selecting an observed sequence from the preattentive group and then defining a Bernoulli process that operated at its average hit rate. From the ensemble of all random sequences that share the observed hit rate, 25 were isolated that by chance also happened to have the same runs  $z$  score as the observed sequence. This number was sufficient to give stable estimates of the average spectra and density differences for the group satisfying the runs constraint. The procedure terminated when 25 matches had been generated for all 209 sequences in the preattentive group. Average density differences and power spectra were then computed over the entire matched ensemble. Figure 17 illustrates the structure that this selection process induces in these statistics. Open circles are averages over the preattentive studies—essentially averages over the curves in Figure 16. Filled circles

<sup>6</sup> The sign of  $\Delta$  makes no difference to this transformation even though it is sensible to associate a spate of short reaction times with a hot streak, implying that  $\Delta$  should be negative. All of the results reported here are invariant under any transformation of the reaction times that preserves the power spectrum.

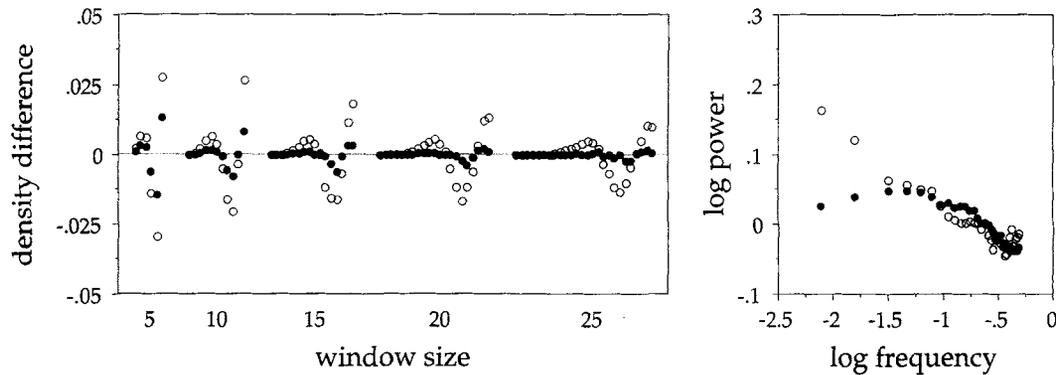


Figure 17. Density differences and power spectra for the subset of random sequences that are matched on hit rate and runs  $z$  score to observed sequences from the preattentive studies. Open circles show average preattentive task data, and filled circles show the ensemble averaged matched sequences.

show averages for random sequences that have the same hit rate and runs  $z$  score distribution as the data. The basic form of the results is as expected: Correlations at small trial scales are constrained to fit the data, but correlations at large scales are absent. In the density differences, this trend appears as nonstationarity at sizes  $N = 5$  and  $N = 10$  (but with smaller amplitude than the data) and convergence to Bernoulli trials as the window size increases. In a similar manner, the spectrum of the matched sequences is tilted at high frequencies (small trial scales) and whitens at low frequencies (large trial scales). It is important to underscore that this example illustrates the maximum effect a runs constraint can produce. In the simulations, only the average runs  $z$  score was constrained—not the entire distribution. So the models may show greater disagreement with the data than the matched sequences depicted here.

The latencies collected for the demonstrations in Gilden (1997) were used as input to the linear probability mapping. The four studies were split into two groups on the basis of their  $1/f$  noise content. The residuals deriving from translation and rotation search have about 40% of their variation in  $1/f$  noise. This percentage is halved in mental rotation and lexical decision. Differences in  $1/f$  content are mapped directly into differences in the runs  $z$  score; consequently, search residuals will naturally create streakier binary sequences than mental rotation or lexical decision. For this reason, the search residuals were isolated from the mental rotation and lexical decision residuals, and two independent simulations were run, each with its own value of  $\Delta$ . It is inevitable that  $\Delta$  (search) must be less than  $\Delta$  (mental rotation, lexical decision) when fit to the same set of observed outcome sequences.

Ensembles of simulated 2AFC outcomes were created by applying the linear probability map to all of the latency residuals contained within a given set of reaction time studies. Each outcome sequence was fabricated by literal interpretation of the probability map; random uniform deviates were scored as hits if they were less than the mapping of a latency residual obtained for a given trial.  $\Delta$  was varied until the ensemble-averaged runs  $z$  score was  $-1$ , matching the value found for preattentive discrimination. The optimal values of  $\Delta$  were unique for a given set of latency residuals and turned out to be  $\Delta$  (mental rotation, lexical decision) = 1 and  $\Delta$  (search) = .65. Once ensembles had been created

that satisfied the runs constraint, density differences and power spectra were computed for each simulated sequence.

Figure 18 shows the medium resolution statistics for simulated outcome sequences generated from the two sets of latency residuals. The agreement between simulated and observed sequences that is evident in this figure is not a trivial result. From a computational standpoint, the agreement is remarkable because the free parameter  $\Delta$  was chosen not to optimize fit on the densities and spectra but, rather, to replicate the observed average runs deficits. As the runs  $z$  score is numerically equal to the Fisher  $Z$  transformation of the serial correlation (lag 1), this simulation demonstrates that once the serial correlation is constrained, the correlations at all scales follow suit. This situation can occur only if discrimination and latency fluctuations are different manifestations of the same fractional Brownian motion. The quantitative agreement between observed data and latency driven 2AFC outcomes represents a convergence of experimental paradigms that used different observers, different stimuli, and different modes of assessment. This is an extreme form of cross-validation, and it implies that the dynamic signified by  $1/f$  noise is not a fiction or an artifact of methodology. Although this dynamic does not yet have a theoretical interpretation, it reveals itself across the fundamental forms of measurement.

## Summary

In this article, I have shown that there is a memory process that is generally active in choice and discrimination. This memory process makes itself known by the presence of  $1/f$  noise in the three major measurement paradigms in psychophysics: speeded judgment, accuracy of discrimination, and production. It is not reducible to sequential priming or any other kind of correlative process that has been invoked in psychological theory. It is not an encoding of experience. Rather, it is the kind of memory that arises in dynamical systems (Beran, 1994) and is an embodiment of how the system moves forward in time. The data presented in this article suggest that  $1/f$  noises are a consequence of an intrinsic dynamic associated with the formation of representations.

The significance of  $1/f$  noise is that it is a very specific kind of memory structure, and it arises in nature only under highly par-

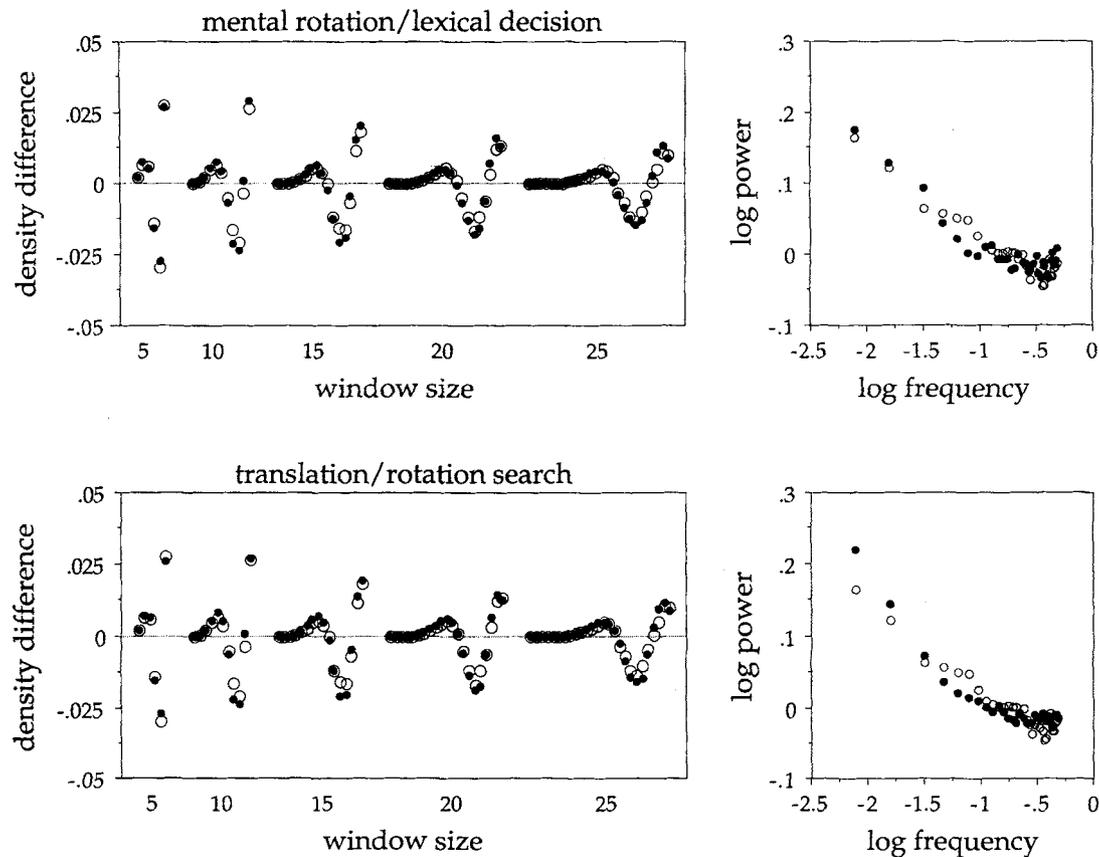


Figure 18. Evaluation of reaction time based simulations of hit rate nonstationarity. Open circles depict the density differences and power spectra averaged over the four preattentive discriminations studies. Filled circles depict models that are based on mental rotation and lexical decision latency residuals (top) and search latency residuals (bottom).

ticularized conditions. Understanding what these conditions are is a central and open problem in theoretical physics. There are many models for the generation of  $1/f$  noise, and they vary in degree of generality as well as in the theoretical constructs they invoke. A common thread that connects a large class of these models is that  $1/f$  noises are byproducts of dynamics that intertwine aspects of order and disorder. These models include random walks in random environments (Marinari, Parisi, Ruelle, & Windey, 1983), tangent bifurcation (Keeler & Farmer, 1986; Pomeau & Manneville, 1980), extremal dynamics (Miller, Miller, & McWhorter, 1993), and self-organized critically (Bak, 1990, 1992; Bak & Chen, 1991; Bak, Chen, & Creutz, 1989; Bak, Tang, & Wiesenfeld, 1987, 1988; Christensen, Olami, & Bak, 1992; Jensen, Christensen, & Fogedby, 1989; Kertesz & Kiss, 1990). This latter theory is unique in the scope of its explanatory power and the nonspecificity of the mechanisms that it requires.

The central idea in the theory of self-organized criticality is that complex systems evolve naturally (self-organization), independent of the particular physics governing the dynamics, to a thermodynamic transition (critical state) that marks the borderline between stability and chaos. In the critical state, a system loses its characteristic temporal and spatial scales with the results that (a) correlations run through the system at all scales and (b) the system emits

fractal structure—structure that has no intrinsic scale. In this view,  $1/f$  noise is the fractal structure in time that signifies the critical state. Self-organized criticality provides a definition for what it means for a system to be complex, it provides an experimental procedure for establishing complexity, and it accounts for the ubiquity of systems in nature that exhibit  $1/f$  noises. As complex cellular automata find application in the study of adapting systems (Ito, 1995; Maslov, Paczuski, & Bak, 1994), the theory becomes relevant to biology and perhaps, eventually, to psychology. For example, it has been shown that the Game of Life (Alstrom & Leao, 1994; Bak et al., 1989) is an example of self-organized critical system. These ideas have also been used in models of spike rate variability (Usher, Stemmler, Koch, & Olami, 1994; Usher, Stemmler, & Olami, 1995). It is interesting that the latter simulations found  $1/f$  noise at the single neuron level but not in ensembles of neurons. Whether the brain does exhibit self-organized criticality is an open question that is of clear relevance given the data presented here.

The finding that cognition generates a dynamical signature as a consequence of its own activity motivates a different perspective on what is signal and what is noise in data. A fair fraction of what experimental psychologists have been calling unexplained variance is literally the engine noise. Its status as unexplained variance

derives mainly from the fact that experimental designs and the ensuing ANOVAs are unable to contemplate any structure not anticipated by the narrow portal on the world offered by a grid of treatment cells. What is in fact occurring in any experimental situation is that responses to stimuli are always attended by a 1/f carrier signal. This signal is loud and present in all paradigms that have the power to reveal it. There is some irony here in that the techniques that have been developed to isolate treatment means so as to consolidate informal theories of mind may in fact be burying one of the most important signatures of what happens when a mind is working.

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